

R.N.G.PATEL INSTITUTE OF TECHNOLOGY-RNGPIT
(An Autonomous Institute U/s UGC Act 1956)

B.Tech. SEMESTER-I, SEMESTER END EXAMINATION – WINTER 2025

SUBJECT CODE: 2SH103

DATE: 15-12-2025

SUBJECT NAME: CALCULUS

TIME: 11:00 AM to 02:00 PM

TOTAL MARKS: 70

Instructions

1. It is **compulsory** for students to write **Enrolment No. /Seat No.** on the question paper.
2. Write answers of **Section A** and **Section B** in **separate answer books**.
3. Attempt all questions from both **Section A** and **Section B**.
4. Each section carries **35 marks**, with a total of **70 marks** for the examination.
5. The figures to the right of each question indicate full marks, make suitable assumptions with justification.
6. BL - Cognitive Level (As per Revised Bloom's Taxonomy) (R-Remember, U-Understanding, A –Application, N –Analyze, E – Evaluate, C -Create), CO - Course Outcomes.

SECTION A

		Marks	BL	CO
Q.1	(a) Solve $\frac{dy}{dx} + y \sin x = e^{\cos x}$.	03	A	3
	(b) Solve $y(xy + 2x^2y^2)dx + x(xy - x^2y^2)dy = 0$.	04	A	3
Q.2	(a) Find $\frac{dy}{dx}$ if $x^3 + y^3 = 3axy$.	03	A	2
	(b) Find the equation of tangent plane and the normal line to the surface $2x^2 + y^2 + 2z = 3$ at the point $(2, 1, -3)$.	04	A	2
	(c) Find the extreme values of the function $x^3 + y^3 - 63(x + y) + 12xy$.	07	A	2
OR				
Q.2	(a) If $u = x^3y + e^{xy^2}$ then prove that $\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial^2 u}{\partial y \partial x}$.	03	A	2
	(b) If $u = \sin^{-1} \left(\frac{x^{1/4} + y^{1/4}}{x^{1/5} + y^{1/5}} \right)$, prove that $x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = \frac{1}{144} \tan u [\tan^2 u - 19]$.	04	A	2
	(c) Find a point on the plane $2x + 3y - z = 5$ which is nearest to the origin.	07	A	2

- Q.3** (a) Form a partial differential equation from $z = (x-2)^2 + (y-3)^2$. **03** **A** **5**
- (b) Solve $yp = 2yx + \log q$. **04** **A** **5**
- (c) Solve $(z-y)p + (x-z)q = y-x$. **07** **A** **5**

OR

- Q.3** (a) Solve $p + q^2 = 1$. **03** **A** **5**
- (b) Form a partial differential equation by eliminating ϕ from
 $\phi(xy + z^2, x + y + z)$. **04** **A** **5**
- (c) Solve $x^2p + y^2q = (x+y)z$. **07** **A** **5**

SECTION B

		Marks	BL	CO
Q.4	(a) Find directional derivative of $\phi = xy^2 + yz^2$ at the point (2,-1,1) in the direction of the vector $\hat{i} + 2\hat{j} + 2\hat{k}$.	03	A	6
	(b) Prove that the vector $\vec{F} = (3x + 2y + 4z)\hat{i} + (2x + 5y + 4z)\hat{j} + (4x + 4y - 8z)\hat{k}$ is both solenoidal and irrotational.	04	A	6
Q.5	(a) Find Eigen values of the matrix $\begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$.	03	A	1
	(b) Find rank of the matrix $\begin{bmatrix} 3 & -2 & 0 & -1 \\ 0 & 2 & 2 & 1 \\ 1 & -2 & -3 & 2 \\ 0 & 1 & 2 & 1 \end{bmatrix}$.	04	A	1
	(c) Solve the following system of equations by Gauss elimination method	07	A	1
	$3x + y - 4z = 13$ $2x - 3y + 7z = 5$ $2x + 19y - 47z = 32$			
OR				
Q.5	(a) Reduce the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 6 & 8 \\ 3 & 4 & 5 \end{bmatrix}$ in row echelon form.	03	A	1
	(b) Solve the following system of equations by Gauss Jordan method	04	A	1
	$x + y + z = 6$ $x + 2y + 3z = 14$ $2x + 4y + 7z = 30$			
(c) Find eigen values and eigen vectors of the following matrix	07	A	1	
	$\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$			

- Q.6** (a) Solve $(D^2 + 9)y = \cos 4x$. **03** **A** **4**
- (b) Solve $y''' - 3y'' + 3y' - y = 4e^t$. **04** **A** **4**
- (c) Using variation of parameters, solve the following equations **07** **A** **4**
 $y'' + a^2y = \tan ax$.

OR

- Q.6** (a) Solve $(D^4 - 1)y = 0$. **03** **A** **4**
- (b) Solve $(D^2 + 2D + 3)y = 2x^2$. **04** **A** **4**
- (c) Using method of undetermined coefficients, solve the following **07** **A** **4**
equations $y'' + 4y = 8x^2$.
