Enrolment No/Seat No.: ____________________

R.N.G.PATEL INSTITUTE OF TECHNOLOGY-RNGPIT

(An Autonomous Institute U/s UGC Act 1956)

B.Tech. SEMESTER-I, SEMESTER END EXAMINATION – WINTER 2025

SUBJECT CODE: 2SH102 **DATE: 15-12-2025**

SUBJECT NAME: MATHEMATICS 1

TIME: 11:00 AM to 02:00 PM **TOTAL MARKS: 70**

Instructions

1. It is **compulsory** for students to write **Enrolment No. /Seat No.** on the question paper.
2. Write answers of **Section A** and **Section B** in **separate answer books**.
3. Attempt all questions from both **Section A** and **Section B**.
4. Each section carries **35 marks**, with a total of **70 marks** for the examination.
5. The figures to the right of each question indicate full marks, make suitable assumptions with justification.
6. BL - Cognitive Level (As per Revised Bloom's Taxonomy) (R-Remember, U-Understanding, A –Application, N –Analyze, E – Evaluate, C -Create), CO - Course Outcomes.

SECTION A

			Marks	BL	CO
Q.1	**(a)**	Solve initial value problem of $3y^2dx + xdy = 0, y(1) = \frac{1}{2}$.	**03**	**A**	**3**
	(b)	Solve $\left(3x^2y^4 + 2xy\right)dx + \left(2x^3y^3 - x^2\right)dy = 0$.	**04**	**A**	**3**
Q.2	**(a)**	Find the equations for tangent plane and normal line at the point $(1,1,1)$ on the surface $x^2 + y^2 + z^2 = 3$.	**03**	**A**	**2**
	(b)	If $u = \tan^{-1}\left(\frac{x^3 + y^3}{x - y}\right)$, prove that $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = \sin 4u - \sin 2u$.	**04**	**A**	**2**
	(c)	Find all the stationary points of the function $f(x,y) = x^3 + 3xy^2 - 15x^2 - 15y^2 + 72x$ after examining whether the function is maximum or minimum at those points.	**07**	**A**	**2**
		OR			
Q.2	**(a)**	If $u = x^3y + e^{xy^2}$ then prove that $\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial^2 u}{\partial y \partial x}$.	**03**	**A**	**2**
	(b)	If $u = f(x - y, y - z, z - x)$, prove that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$.	**04**	**A**	**2**

	(c)	Find the extreme values of the function $x^3+y^3-63(x+y)+12xy$.	**07**	**A**	**2**
Q.3	**(a)**	Find the directional derivative of $\phi = xy^2+yz^2$ at point $(2,-1,1)$ in the direction of the vector $\hat{i}+2\hat{j}+2\hat{k}$.	**03**	**A**	**6**
	(b)	Determine the constant a and b such that curl of $\overline{F}=(2xy+3yz)\hat{i}+(x^2+axz-4z^2)\hat{j}+(3xy+2byz)\hat{k}$, is zero.	**04**	**A**	**6**
	(c)	Show that $\overline{F}=(6xy+z^3)\hat{i}+(3x^2-z)\hat{j}+(3xz^2-y)\hat{k}$, is irrotational vector and hence find its scalar potential.	**07**	**A**	**6**

OR

Q.3	**(a)**	If $\overline{A}=(x^2z)\hat{i}-2(y^3z^2)\hat{j}+(xy^2z)\hat{k}$, find $\nabla.\overline{A}$ at the point $(1,-1,1)$.	**03**	**A**	**6**
	(b)	Prove that vector $\overline{F}=(y^2-z^2+3yz-2x)\hat{i}+(3xz+2xy)\hat{j}+(3xy-2xz+2z)\hat{k}$, is both solenoidal and irrotational.	**04**	**A**	**6**
	(c)	If $\overline{F}=2xyz\,\hat{i}+(x^2z+2y)\hat{j}+x^2y\,\hat{k}$, then I) If $\overline{F}$ is conservative, find its scalar potential ϕ II) Find the work done in moving a particle under this force field from (0,1,1) to (1,2,0).	**07**	**A**	**6**

SECTION B

			Marks	BL	CO
Q.4	(a)	Evaluate $\int_0^1\int_0^1 \frac{dxdy}{\sqrt{(1-x^2)(1-y^2)}}$.	03	A	5
	(b)	Change the order of integration and evaluate $\int_0^\infty\int_x^\infty \frac{e^{-y}}{y}dydx$.	04	A	5
Q.5	(a)	Find rank of the matrix $\begin{bmatrix}-2 & 1 & 3\\ 1 & 4 & 5\\ 0 & 1 & 2\end{bmatrix}$.	03	A	1
	(b)	Find eigen values and eigen vectors of the matrix $\begin{bmatrix}5 & 4\\ 1 & 2\end{bmatrix}$.	04	A	1
	(c)	Solve the following system of equations by Gauss Elimination method $4x-2y+6z=8$ $x+y-3z=-1$ $15x-3y+9z=21$	07	A	1

OR

			Marks	BL	CO
Q.5	(a)	Find rank of the matrix $\begin{bmatrix}1 & 2 & -1 & 3\\ 3 & 4 & 0 & -1\\ -1 & 0 & -2 & 7\end{bmatrix}$.	03	A	1
	(b)	Find inverse of the matrix $\begin{bmatrix}1 & 0 & 1\\ -1 & 1 & 1\\ 0 & 1 & 0\end{bmatrix}$ by Gauss Jordan method.	04	A	1
	(c)	Find eigen values and eigen vectors of the matrix $\begin{bmatrix}1 & 0 & -1\\ 1 & 2 & 1\\ 2 & 2 & 3\end{bmatrix}$.	07	A	1
Q.6	(a)	Solve: $\left(D^2-9\right)y=0, y\left(0\right)=2, y'\left(0\right)=-1$.	03	A	4
	(b)	Solve: $\left(D^2+D\right)y=x^2+2x+4$.	04	A	4
	(c)	Using variation of parameters, solve the equation $\left(D^2+1\right)y=\operatorname{cosec} x$.	07	A	4

OR

Q.6	**(a)**	Solve: $y'' - 4y' + 4y = 0, y(0) = 3, y'(0) = 1$.	**03**	**A**	**4**
	(b)	Solve: $\left(D^2 + 3D + 2\right)y = \sin 2x$.	**04**	**A**	**4**
	(c)	Using method of undetermined coefficients, solve the equation $y'' + 4y = 8x^2$.	**07**	**A**	**4**

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