*Enrolment No/Seat No.:* ____________________

# R.N.G.PATEL INSTITUTE OF TECHNOLOGY-RNGPIT

***(An Autonomous Institute U/s UGC Act 1956)***

**IMSc-IT SEMESTER-I, SEMESTER END EXAMINATION – WINTER 2025**

**SUBJECT CODE: 2BS102** **DATE: 29-12-2025**

**SUBJECT NAME: FUNDAMENTAL OF MATHEMATICS**

**TIME: 11:00 AM to 02:00 PM** **TOTAL MARKS: 70**

**Instructions**

1. It is **compulsory** for students to write **Enrolment No. /Seat No.** on the question paper.
2. Write answers of **Section A** and **Section B** in **separate answer books**.
3. Attempt all questions from both **Section A** and **Section B**.
4. Each section carries **35 marks**, with a total of **70 marks** for the examination.
5. The figures to the right of each question indicate full marks, make suitable assumptions with justification.
6. BL - Cognitive Level (As per Revised Bloom's Taxonomy) (R-Remember, U-Understanding, A –Application, N –Analyze, E – Evaluate, C -Create), CO - Course Outcomes.

## SECTION A

| | | | Marks | BL | CO |
|---|---|---|---|---|---|
| **Q.1** | **(a)** | Find cofactor of the following: $\begin{vmatrix} 1 & 3 & 2 \\ 3 & 1 & 2 \\ 1 & 1 & 3 \end{vmatrix}$ | **03** | **A** | **3** |
| | **(b)** | Solve $\begin{matrix} 2x-3y=7 \\ 7x-3y=10 \end{matrix}$ by using Cramer's rule. | **04** | **A** | **3** |
| **Q.2** | **(a)** | Find the characteristic equation of the matrix $\begin{bmatrix} 4 & 2 & -2 \\ -5 & 3 & 2 \\ -2 & 4 & 1 \end{bmatrix}$. | **03** | **A** | **4** |
| | **(b)** | Verify Cayley Hamilton Theorem for the matrix $\begin{bmatrix} 8 & 5 & 5 \\ 0 & 3 & 0 \\ 5 & 5 & 8 \end{bmatrix}$. | **04** | **A** | **4** |
| | **(c)** | Find eigenvalues and eigen vectors of the matrix $\begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}$. | **07** | **A** | **4** |
| | | **OR** | | | |
| **Q.2** | **(a)** | Show that the matrix $A = \frac{1}{2}\begin{bmatrix} 1+i & 1-i \\ 1-i & 1+i \end{bmatrix}$ is Unitary. | **03** | **A** | **4** |
| | **(b)** | Find eigenvalues and eigen vectors of the matrix $\begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$. | **04** | **A** | **4** |
| | **(c)** | Let $A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$. Find matrix $P$ such that $P^{-1}AP$ is diagonal matrix. | **07** | **A** | **4** |

| | | | Marks | BL | CO |
|---|---|---|---|---|---|
| **Q.3** | **(a)** | Find the limits following:<br>i) $\lim_{x\to 1}\frac{(x^2-2x+1)}{x-1}$<br>ii) $\lim_{x\to 0}\frac{(1+x)^n-1}{x}$ | **03** | **R,A** | **5** |
| | **(b)** | Find derivative of functions:<br>i) $f(x)=\sin x^2+\log x^3$<br>ii) $\frac{d}{dx}[log(x^2-4x+2)]$ | **04** | **A** | **5** |
| | **(c)** | Check whether the function given below is continuous or not?<br>$f(x)=\begin{cases} x+2, & if\ x<3 \\ 5, & if\ x=3 \\ x^2-4, & if\ x>3 \end{cases}$ | **07** | **A** | **5** |

**OR**

| | | | | | |
|---|---|---|---|---|---|
| **Q.3** | **(a)** | Find the limits following:<br>i) $\lim_{x\to 0}\frac{e^x-1}{x}$<br>ii) $\lim_{x\to 1}\frac{\cos x-1}{x}$ | **03** | **A** | **5** |
| | **(b)** | Find derivative of functions:<br>i) $f(x)=e^x \sin x$<br>ii) $f(x)=\frac{x+1}{e^x}$ | **04** | **A** | **5** |
| | **(c)** | Check whether the function given below is continuous or not?<br>$f(x)=\begin{cases} x+2, & if\ x<1 \\ 1\ , & if\ x=1 \\ 2-x, & if\ x>1 \end{cases}$ | **07** | **A** | **5** |

## SECTION B

| | | | Marks | BL | CO |
|---|---|---|---|---|---|
| **Q.4** | **(a)** | State and prove Distributive Law for set theory. | **03** | **A** | **1** |
| | **(b)** | Show that $f:R\to R$ defined by $f(x)=7x-11$ is one-to-one and onto. | **04** | **A** | **1** |
| **Q.5** | **(a)** | Reduce the matrix $\begin{bmatrix} 0 & 6 & 7 \\ -5 & 4 & 2 \\ 1 & -2 & 0 \end{bmatrix}$ into row echelon (upper triangular) form. | **03** | **A** | **2** |
| | **(b)** | Find inverse of the matrix $A=\begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 3 \\ 1 & 0 & 8 \end{bmatrix}$ by using Gauss-Jordan method. | **04** | **A** | **2** |
| | **(c)** | Solve the system of equations<br>$2x+3y+4z=11$<br>$x+5y+7z=15$<br>$3x+11y+13z=25$ | **07** | **A** | **2** |

**OR**

| | | | | | |
|---|---|---|---|---|---|
| **Q.5** | **(a)** | Reduce the matrix $\begin{bmatrix} -2 & 1 & 3 \\ 1 & 4 & 5 \\ 0 & 1 & 2 \end{bmatrix}$ into unit matrix by using elementary row operations. | **03** | **A** | **2** |
| | **(b)** | Express the matrix $A = \begin{bmatrix} 1 & 5 & 7 \\ -1 & -2 & -4 \\ 8 & -2 & 13 \end{bmatrix}$ as a sum of symmetric matrix and skew symmetric matrix. | **04** | **A** | **2** |
| | **(c)** | Test for consistency and solve<br>$x+y+z=6$<br>$2x-y+3z=14$<br>$x+4y-z=2$ | **07** | **A** | **2** |
| **Q.6** | **(a)** | State Rolle's theorem and verify it for $y = x^2 + 1$ in $[-1,1]$. | **03** | **A** | **6** |
| | **(b)** | Find Maclaurin's series of $e^x$. | **04** | **A** | **6** |
| | **(c)** | Find the extreme values of the function $x^2 + y^2 - 4x - 6y + 13$. | **07** | **A** | **6** |

**OR**

| | | | | | |
|---|---|---|---|---|---|
| **Q.6** | **(a)** | Show that $f(x) = \sqrt{25 - x^2}$ satisfies the hypothesis of mean value theorem on [3, 5] and find all values of $c$ in (3, 5) that satisfy the conclusion of the theorem. | **03** | **A** | **6** |
| | **(b)** | Expand $7 + (x + 2) + 3(x + 2)^3 + (x + 2)^4$ in powers of $(x - 1)$. | **04** | **A** | **6** |
| | **(c)** | Find the extreme values of the function $x^3 + y^3 - 3x - 12y + 20$. | **07** | **A** | **6** |

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