

R.N.G.PATEL INSTITUTE OF TECHNOLOGY-RNGPIT
(An Autonomous Institute U/s UGC Act 1956)

B.Tech. SEMESTER-I, SEMESTER END EXAMINATION – WINTER 2025

SUBJECT CODE: 1SH103

DATE: 15-12-2025

SUBJECT NAME: CALCULUS

TIME: 11:00 AM to 02:00 PM

TOTAL MARKS: 70

Instructions

1. It is **compulsory** for students to write **Enrolment No. /Seat No.** on the question paper.
2. Write answers of **Section A** and **Section B** in **separate answer books**.
3. Attempt all questions from both **Section A** and **Section B**.
4. Each section carries **35 marks**, with a total of **70 marks** for the examination.
5. The figures to the right of each question indicate full marks, make suitable assumptions with justification.
6. BL - Bloom's Taxonomy Levels (R-Remember, U-Understanding, A –Application, N –Analyze, E – Evaluate, C -Create), CO - Course Outcomes.

SECTION A

Marks BL CO

Q.1 Multiple-Choice Questions

[05]

(a) If $f = x^2 + y^2$, then $\frac{\partial^2 f}{\partial x \partial y} = \underline{\hspace{2cm}}$.

1 A 2

(i) 1

(ii) 0

(iii) 2

(iv) y

(b) If $u = x^2y + y^2z + z^2y$, then $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = \underline{\hspace{2cm}}$.

1 A 2

(i) $2xy + 2yz + 2zx$

(ii) $(x + y + z)^2$

(iii) $\frac{1}{x + y + z}$

(iv) $x^2 + y^2 + z^2$

(c) Which one of the following is a non-linear PDE?

1 U 4

(i) $u_x + u_y = u^2$

(ii) $u_x + u_y = x + y$

(iii) $u_x \cdot u_y = 1$

(iv) $u_{xx} + u_{yy} = 0$

(d) If \vec{F} is conservative, then _____.

1 R 5

(i) $\nabla \times \vec{F} = 0$

(ii) $\nabla \times \vec{F} \neq 0$

(iii) $\nabla \bar{F} = 0$

(iv) $\nabla \cdot \bar{F} = 0$

(e) If $\phi = xyz$, then the value of $|\text{grad } \phi|$ at $(1, 2, -1)$ is

1 A 5

(i) 0

(ii) 1

(iii) 2

(iv) 3

Q.2 Attempt Any Two

[10]

(a) If $u = f(x - y, y - z, z - x)$, then show that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$.

5 A 2

(b) If $u = \sin^{-1} \left(\frac{x^{1/4} + y^{1/4}}{x^{1/6} + y^{1/6}} \right)$, prove

5 A 2

$$x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = \frac{1}{144} \tan u [\tan^2 u - 11].$$

(c) (i) Find the equations of the tangent plane and normal lines to the surface $2x^2 + y^2 + 2z = 3$ at $(2, 1, -3)$.

5 A 2

(ii) If $x^y + y^x = c$, find $\frac{dy}{dx}$.

Q.3 Attempt Any Two

[10]

(a) Solve $yp = 2yx + \log q$.

5 A 4

(b) Solve $x^2(y - z)p + y^2(z - x)q = z^2(x - y)$.

5 A 4

(c) Eliminate the function from the relation $f(xy + z^2, x + y + z) = 0$.

5 A 4

Q.4 Attempt Any Two

[10]

(a) (i) Find the directional derivative of $\phi = xy^2 + yz^2$ at point $(2, -1, 1)$ in the direction of the vector $\hat{i} + 2\hat{j} + 2\hat{k}$.

5 A 5

(ii) If $\bar{A} = (ax^2y + yz)\hat{i} + (xy^2 - xz^2)\hat{j} + (2xyz - 2x^2y^2)\hat{k}$ is solenoidal, find the constant a .

(b) A vector field is given $\bar{F} = (x^2 - y^2 - x)\hat{i} - (2xy + y)\hat{j}$. Show that the field is irrotational and find its scalar potential.

5 A 5

(c) Prove that $\int_C \bar{F} \cdot d\bar{r} = 3\pi$, where $\bar{F} = z\hat{i} + x\hat{j} + y\hat{k}$ and C is the arc of the curve

5 A 5

$$\bar{r} = \cos t \hat{i} + \sin t \hat{j} + t \hat{k} \text{ from } t = 0 \text{ to } t = 2\pi.$$

SECTION B

Marks BL CO

Q.5 Multiple-Choice Questions

[05]

- (a) The rank of a matrix A is equal to _____ 1 U 1
- (i) The number of rows of the the matrix A
 (ii) The number of columns of the matrix
 (iii) The number of rows and columns of the matrix A
 (iv) none of these
- (b) The characteristics equation of the matrix $A = \begin{bmatrix} 14 & -10 \\ 5 & 1 \end{bmatrix}$ is _____. 1 U 1
- (i) $\lambda^2 + 13\lambda - 36 = 0$ (ii) $\lambda^2 - 13\lambda - 36 = 0$
 (iii) $\lambda^2 - 13\lambda + 36 = 0$ (iv) $\lambda^2 + 13\lambda + 36 = 0$
- (c) A system of linear equations represented by the matrix equation $AX = B$ will be consistent if _____. 1 A 3
- (i) $\rho(A) = \rho(A : B)$ (ii) $\rho(A) \neq \rho(A : B)$
 (iii) $\rho(A) < \rho(A : B)$ (iv) $\rho(A) > \rho(A : B)$
- (d) The solution of the equation $\frac{d^2y}{dx^2} - y = 0$ is _____. 1 A 3
- (i) $y = (c_1 + c_2x)e^{-x}$ (ii) $y = c_1e^{-x} + c_2e^x$
 (iii) $y = e^x (c_1 \cos x + c_2 \sin x)$ (iv) $y = c_1 \cos x + c_2 \sin x$
- (e) The integrating factor of $\frac{dy}{dx} + y = e^{-x}$ is _____. 1 A 3
- (i) e^x (ii) e^{-x}
 (iii) $\log x$ (iv) x

Q.6 Attempt Any Two

[10]

- (a) Find the eigen values and eigen vectors of the matrix $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & -3 & 3 \end{bmatrix}$. 5 A 1

(b) Find the rank of the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 6 & 8 \\ 3 & 4 & 5 \end{bmatrix}$. 5 A 1

(c) Investigate for what values of λ and μ the equations 5 A 1

$$x + 2y + z = 8$$

$$2x + 2y + 2z = 13$$

$$3x + 4y + \lambda z = \mu$$

have (i) no solution (ii) a unique solution, and (iii) many solutions.

Q.7 Attempt Any Two [10]

(a) Solve $(x^3 + 3xy^2)dx + (3x^2y + y^3)dy = 0$. 5 A 3

(b) Solve $\frac{dy}{dx} + y \sin x = e^{\cos x}$. 5 A 3

(c) Solve $\frac{dy}{dx} + \frac{y}{x} = \frac{y^2}{x^2}$. 5 A 3

Q.8 Attempt Any Two [10]

(a) Solve $(D^2 - 4D + 4)y = 0, y(0) = 3, y'(0) = 1$. 5 A 3

(b) Solve $(D^3 + 8)y = x^4 + 2x + 1$. 5 A 3

(c) By using the method of variation of parameters, solve $y'' - 3y' + 2y = e^x$. 5 A 3
