Enrolment No/Seat No.: ____________________

R.N.G.PATEL INSTITUTE OF TECHNOLOGY-RNGPIT

(An Autonomous Institute U/s UGC Act 1956)

B.Tech. SEMESTER-I, SEMESTER END EXAMINATION – WINTER 2025

SUBJECT CODE: 1SH102 **DATE: 15-12-2025**

SUBJECT NAME: MATHEMATICS-I

TIME: 11:00 AM to 02:00 PM **TOTAL MARKS: 70**

Instructions

1. It is **compulsory** for students to write **Enrolment No. /Seat No.**on the question paper.
2. Write answers of**Section A** and **Section B** in **separate answer books**.
3. Attempt all questions from both **Section A** and **Section B**.
4. Each section carries **35 marks**, with a total of **70 marks** for the examination.
5. The figures to the right of each question indicate full marks, make suitable assumptions with justification.
6. BL - Bloom's Taxonomy Levels (R-Remember, U-Understanding, A –Application, N –Analyze, E –Evaluate, C -Create), CO - Course Outcomes.

SECTION A

		Marks	BL	CO
Q.1	**Multiple-Choice Questions**	**[05]**		
	(a) If $u = x^2y + y^2z + z^2x$, then $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} =$ ______ . **(i)** $3(x^2y + y^2z + z^2x)$ **(ii)** $(x+y+z)^2$ **(iii)** $\frac{1}{x+y+z}$ **(iv)** 0	**1**	**R**	**2**
	(b) If $f = x^2 + y^2$, then $\frac{\partial^2 f}{\partial x \partial y} =$ ___ **(i)**1 **(ii)**0 **(iii)**2 **(iv)**y	**1**	**U**	**2**
	(c) The region $\int_1^2 \int_1^2 dx\, dy$ represents **(i)**rectangle **(ii)**square **(iii)**circle **(iv)**triangle	**1**	**R**	**4**
	(d) If $\bar{F}$ is conservative then **(i)** $\nabla \times \bar{F} = 0$ **(ii)** $\nabla \times \bar{F} \neq 0$ **(iii)** $\nabla \bar{F} = 0$ **(iv)** $\nabla \cdot \bar{F} = 0$	**1**	**U**	**5**

	(e) If $\nabla\phi$ is solenoidal then $\nabla^2\phi =$____ **(i)**1 **(ii)**0 **(iii)**-1 **(iv)**none of these	**1**	**U**	**5**
Q.2	**Attempt Any Two**	**[10]**		
	(a) If $z = x + y^x$, prove that $\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial^2 z}{\partial y \partial x}$.	**5**	**A**	**2**
	(b) Find extreme values of the function $f(x, y) = x^3 + y^3 - 3x - 12y + 20$.	**5**	**A**	**2**
	(c) If $u = f\left(\frac{y-x}{xy}, \frac{z-x}{xz}\right)$, show that $x^2 \frac{\partial u}{\partial x} + y^2 \frac{\partial u}{\partial y} + z^2 \frac{\partial u}{\partial z} = 0$.	**5**	**A**	**2**
Q.3	**Attempt Any Two**	**[10]**		
	(a) Change the order of integration and evaluate $\int_0^\infty \int_x^\infty \frac{e^{-y}}{y} dydx$.	**5**	**A**	**4**
	(b) Evaluate $\iint y\, dx\, dy$ over the region enclosed by the parabola $x^2 = y$ and line $y = x + 2$.	**5**	**A**	**4**
	(c) Evaluate $\iint (x^2 + y^2) dxdy$ over the region bounded by the lines $y = 4x, x + y = 3, y = 0, y = 2$.	**5**	**A**	**4**
Q.4	**Attempt Any Two**	**[10]**		
	(a) If $\bar{F} = 3xy\hat{i} - y^2\hat{j}$, evaluate $\int_C \bar{F} \cdot d\bar{r}$ where C is the arc of parabola $y = 2x^2$ from *(0,0)* to *(1,2)*.	**5**	**A**	**5**
	(b) Using Green's theorem, evaluate $\oint_C (x - y) dx + (x + y) dy$ where C is region enclosed by $y = x^2$ and $y^2 = x$.	**5**	**A**	**5**
	(c) Prove that $\bar{F} = (y^2 - z^2 + 3yz - 2x)\hat{i} + (3xz + 2xy)\hat{j} + (3xy - 2xz + 2z)\hat{k}$ is both solenoidal and irrotational.	**5**	**A**	**5**

SECTION B

		Marks	BL	CO
Q.5	**Multiple-Choice Questions**	**[05]**		
	(a) For the matrix $A=\begin{bmatrix}1 & 2\\ 1 & 2\end{bmatrix}$ the rank is ______ **(i)** 0 **(ii)** 1 **(iii)** 2 **(iv)** None of these	**1**	**A**	**1**
	(b) The Eigen value of the matrix is $\begin{bmatrix}2 & 0\\ 2 & 3\end{bmatrix}$ is_____ **(i)** 2,1 **(ii)** 2,2 **(iii)** 2, 3 **(iv)** None of these	**1**	**A**	**1**
	(c) The degree and order of $\left(\frac{d^3y}{dx^3}\right)^4+\left(\frac{dy}{dx}\right)^2+y=0$ is ______. **(i)** 3, 4 **(ii)** 3,1 **(iii)** 1, 2 **(iv)** 2,3	**1**	**A**	**3**
	(d) The integrating factor of $\frac{dy}{dx}+\frac{1}{x}y=e^x$ is ______. **(i)** e^x **(ii)** e^{-x} **(iii)** $\log x$ **(iv)** x	**1**	**A**	**3**
	(e) The roots of $\left(D^2+2D+1\right)y=0$ is ______. **(i)** real and distinct **(ii)** real and repeated **(iii)** complex **(iv)** None of these	**1**	**R**	**3**
Q.6	**Attempt Any Two**	**[10]**		
	(a) Find rank of the matrix $\begin{bmatrix}-2 & 1 & 3\\ 1 & 4 & 5\\ 0 & 1 & 2\end{bmatrix}$.	**5**	**A**	**1**
	(b) Find Eigen value and Eigen vector of the matrix $\begin{bmatrix}5 & 4\\ 1 & 2\end{bmatrix}$.	**5**	**A**	**1**

(c) Solve the system of equations by Gauss elimination method $x+y+2z=9$, $2x+4y-3z=1$, $3x+6y-5z=0$		**5**	**A**	**1**

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Q.7 Attempt Any Two **[10]**

(a) Solve: $\left(x^4-2xy^2+y^4\right)dx-\left(2x^2y-4xy^3+\sin y\right)dy=0$.	**5**	**A**	**3**
(b) Solve: $\frac{dy}{dx}+2y\tan x=\sin x$.	**5**	**A**	**3**
(c) Solve: $\frac{dy}{dx}+\frac{2y}{x}=x^2y^2$.	**5**	**A**	**3**

Q.8 Attempt Any Two **[10]**

(a) Solve: $\left(D^2-9\right)y=0, y\left(0\right)=2, y'\left(0\right)=-1$.	**5**	**A**	**3**
(b) Solve: $\left(D^2+16\right)y=x^4+e^{3x}+\cos 3x$.	**5**	**A**	**3**
(c) Using method of Undetermined coefficients, solve the following equations $y''+2y'+10y=25x^2+3$.	**5**	**A**	**3**

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