

R.N.G.PATEL INSTITUTE OF TECHNOLOGY-RNGPIT
(An Autonomous Institute U/s UGC Act 1956)

IMSc-IT SEMESTER-III, SEMESTER END EXAMINATION – WINTER 2025

SUBJECT CODE: 1BS301

DATE: 16-12-2025

SUBJECT NAME: NUMERICAL METHODS

TIME: 11:00 AM to 02:00 PM

TOTAL MARKS: 70

Instructions

1. It is **compulsory** for students to write **Enrolment No. /Seat No.** on the question paper.
2. Write answers of **Section A** and **Section B** in **separate answer books**.
3. Attempt all questions from both **Section A** and **Section B**.
4. Each section carries **35 marks**, with a total of **70 marks** for the examination.
5. The figures to the right of each question indicate full marks, make suitable assumptions with justification.
6. BL - Cognitive Level (As per Revised Bloom's Taxonomy) (R-Remember, U-Understanding, A –Application, N –Analyze, E – Evaluate, C -Create), CO - Course Outcomes.

SECTION A

- | | Marks | BL | CO | | | | | | | | | | | | |
|---|-----------|----------|----------|--------|--------|------|--------------|--------|--------|--------|--------|--------|--|--|--|
| Q.1 (a) Perform three iteration of Gauss Jacobi method to solve the system of equations: | 03 | A | 6 | | | | | | | | | | | | |
| $10x + y + z = 6$ $x + 10y + z = 6$ $x + y + 10z = 6$ | | | | | | | | | | | | | | | |
| (b) Solve the following system of equations, by the Gauss–Seidel method: | 04 | A | 6 | | | | | | | | | | | | |
| $5x + y - z = 10$ $2x + 4y + z = 14$ $x + 5y + 8z = 20$ | | | | | | | | | | | | | | | |
| Q.2 (a) Certain corresponding values of x and $\log_{10} x$ are (300,2.4771), (304,2.4829), (305,2.4843) and (307,2.4871). Find. $\log_{10} 301$ | 03 | A | 3 | | | | | | | | | | | | |
| (b) Find the value of $\tan^{-1} 0.26$ by using Newton's backward interpolation formula for the following table: | 04 | A | 3 | | | | | | | | | | | | |
| <table border="1" style="margin-left: auto; margin-right: auto;"> <tr> <td style="padding: 5px;">x</td> <td style="padding: 5px;">0.10</td> <td style="padding: 5px;">0.15</td> <td style="padding: 5px;">0.20</td> <td style="padding: 5px;">0.25</td> <td style="padding: 5px;">0.30</td> </tr> <tr> <td style="padding: 5px;">$y = \tan x$</td> <td style="padding: 5px;">0.1003</td> <td style="padding: 5px;">0.1511</td> <td style="padding: 5px;">0.2027</td> <td style="padding: 5px;">0.2553</td> <td style="padding: 5px;">0.3093</td> </tr> </table> | x | 0.10 | 0.15 | 0.20 | 0.25 | 0.30 | $y = \tan x$ | 0.1003 | 0.1511 | 0.2027 | 0.2553 | 0.3093 | | | |
| x | 0.10 | 0.15 | 0.20 | 0.25 | 0.30 | | | | | | | | | | |
| $y = \tan x$ | 0.1003 | 0.1511 | 0.2027 | 0.2553 | 0.3093 | | | | | | | | | | |
| (c) By Lagrange's interpolation formula, find the value of y when $x = 10$ from the following table: | 07 | A | 3 | | | | | | | | | | | | |
| <table border="1" style="margin-left: auto; margin-right: auto;"> <tr> <td style="padding: 5px;">x</td> <td style="padding: 5px;">5</td> <td style="padding: 5px;">6</td> <td style="padding: 5px;">9</td> <td style="padding: 5px;">11</td> </tr> <tr> <td style="padding: 5px;">y</td> <td style="padding: 5px;">12</td> <td style="padding: 5px;">13</td> <td style="padding: 5px;">14</td> <td style="padding: 5px;">16</td> </tr> </table> | x | 5 | 6 | 9 | 11 | y | 12 | 13 | 14 | 16 | | | | | |
| x | 5 | 6 | 9 | 11 | | | | | | | | | | | |
| y | 12 | 13 | 14 | 16 | | | | | | | | | | | |
| OR | | | | | | | | | | | | | | | |
| Q.2 (a) Construct the divided difference table with the arguments 2, 4, 9, 10 of the function $f(x) = x^3 - 2x$. | 03 | A | 3 | | | | | | | | | | | | |

- (b) Find the cubic polynomial and find the value of $y(8)$ from the following data: **04 A 3**

x	1	3	5	7
y	24	120	336	720

- (c) Find the Lagrange interpolating polynomial from the following data: **07 A 3**

x	0	1	4	5
y	1	3	24	39

- Q.3** (a) Use Euler's method to solve $\frac{dy}{dx} = -y$ with $y(0) = 1$ at $x = 0.3$ **03 A 5**
- (b) Given the differential equation $y' = 1 + xy$ with the initial condition $y(0) = 1$, Obtain the Taylor's series for $y(x)$ and compute $y(0.2)$ correct up to four decimal places. **04 A 5**
- (c) Apply Runge-Kutta method of order two to calculate $y(0.1)$ and $y(0.2)$, given that $\frac{dy}{dx} = y - x$ with $y(0) = 2$ taking step size $h = 0.1$. **07 A 5**

OR

- Q.3** (a) Using modified Euler's method, find $y(0.1)$ from the initial value problem $y' = xy$, $y(0) = 1$ by taking step size $h = 0.1$ **03 A 5**
- (b) Solve $y'' - xy - y = 0$ with the condition $y(0) = 1$ and $y'(0) = 0$ using Taylor's series method. Also determine the value of $y(0.1)$. **04 A 5**
- (c) Using Runge-Kutta method of fourth order, solve $\frac{dy}{dx} = x + \sqrt{y}$ where $y(0) = 4$ at $x = 0.1$ taking step size $h = 0.1$. **07 A 5**

SECTION B

Marks BL CO

- Q.4 (a)** Apply least square method fit the best straight line to the data: **03** **A** **1**

x	1	2	3	4	5
y	2	3	5	7	8

- (b)** A company launched a marketing campaign and observed the number of logins over the next few days. Fit a curve of the form $y = ae^{bx}$: **04** **A** **1**

Day	1	2	3	4	5	6	7
Numbers of Login	200	300	440	600	810	1080	1450

- Q.5 (a)** Derive iteration formula for \sqrt{N} and hence find $\sqrt{3}$ by Newton-Raphson method. **03** **A** **2**
- (b)** Find a real root of the equation $2x - \log_{10} x = 7$ by regula falsi method. **04** **A** **2**
- (c)** Find the positive root of $x^3 - 2x - 5$, correct up to two decimal places by bisection method. **07** **A** **2**

OR

- Q.5 (a)** Perform the three iterations of the bisection method to obtain a root of the equation $f(x) = x^3 - x - 1 = 0$. **03** **A** **2**
- (b)** Find a real root of the equation $x^4 - x^3 + 10x + 7 = 0$, correct up to three decimal places between -2 and -1 by the Newton-Raphson method. **04** **A** **2**
- (c)** Solve $xe^x - 1 = 0$ correct up to three decimal places between 0 and 1 by secant method. **07** **A** **2**

- Q.6 (a)** What is numerical differentiation? State the Newton's Forward Difference Formula for the first derivative $(\frac{dy}{dx})$ and second derivative $(\frac{d^2y}{dx^2})$. **03** **R** **4**

- (b)** The distances (x cm) traversed by a particle at different times (t seconds) are given below: **04** **A** **4**

x	0.0	0.1	0.2	0.3	0.4	0.5	0.6
y	3.01	3.16	3.29	3.36	3.40	3.38	3.31

Find the velocity and acceleration of the particle at $t = 0.3$ second.

- (c)** Evaluate **07** **A** **4**

$$\int_0^1 e^x dx$$

by using Trapezoidal rule and Simpson's $\frac{1}{3rd}$ rule (take $h=0.1$)

OR

- Q.6 (a)** Write formula of the following: **03** **R** **4**
- i) Trapezoidal rule

ii) Simpson's 1/3 rule

iii) Simpson's 3/8 rule

- (b) In computer systems, the performance of a processor can be measured by its execution time (in seconds) for processing a given data size (in MB): **04 A 4**

Data size x (MB)	10	20	30	40	50
Execution Time t (s)	2.0	2.9	4.1	5.6	7.5

We want to find how fast the execution time is increasing with respect to the data size *i. e.* the rate of change of execution time ($\frac{dt}{dx}$) at the beginning for $x = 10$ MB data.

- (c) Evaluate **07 A 4**

$$\int_0^1 \frac{1}{1+x^2} dx$$

by using Trapezoidal rule and Simpson's $\frac{3}{8}$ rule by taking $h = \frac{1}{6}$.
