R.N.G.PATEL INSTITUTE OF TECHNOLOGY-RNGPIT (An Autonomous College U/s UGC Act 1956)

B.Tech. SEMESTER-I, SEMESTER END EXAMINATION – SUMMER 2025Subject Code: 1SH103Date: 02-06-2025Subject Name: CalculusTotal Marks: 70Time: 11:00 AM to 02:00 PMTotal Marks: 70

Instructions

- 1. It is compulsory for students to write Enrolment No. /Seat No. on the question paper.
- 2. Write answers of Section A and Section B in separate answer books.
- 3. Attempt all questions from both Section A and Section B.
- 4. Each section carries **35 marks**, with a total of **70 marks** for the examination.
- 5. The figures to the right of each question indicate full marks, make suitable assumptions with justification.
- 6. BL Bloom's Taxonomy Levels (R-Remember, U-Understanding, A –Application, N –Analyze, E Evaluate, C -Create), CO Course Outcomes.

SECTION A

			Marks	BL	CO
Q.1	Multiple-Choice Questions		[05]		
	(a) If $f(x, y) = c$ be an implicit function	then $\frac{dy}{dx} =$	1	R	2
	$(\mathbf{i}) - \frac{f_y}{f_x}$	(ii) $-\frac{f_x}{f_y}$			
	(iii) $-\frac{p}{q}$	$(\mathbf{iv}) - \frac{q}{p}$			
	(b) The modified Euler's theorem involv	es:	1	U	2
	(i)Partial derivatives only	(ii)Second-order partial derivatives			
	(iii)Total derivative of a homogeneous function	(iv)Higher-order homogeneous function	1	D	1
	(c) The definite integral $\int_a f(x) dx$ represented by $\int_a f(x) dx$	esents:	1	N	4
	(i) The slope of the function $f(x)$	(ii)The volume under the surface			
	(iii)The value of $f(b) + f(a)$	(iv)The area under the curve from a to b			
	(d) A vector field \overline{F} is said to be a source	if	1	R	5
	$(\mathbf{i})div\ \overline{F}=0$	$(\mathbf{ii})div \ \overline{F} > 0$			
	(iii) $div \bar{F} < 0$	(iv) $div \bar{F} \neq 0$			

(e) A vector field \overline{F} is path-independent if and only if

(i)
$$\nabla \cdot \overline{F} = 0$$

(ii) $\nabla \times \overline{F} = 0$ in simply connected
region
(iii) $\overline{F} \cdot d\overline{r} = 0$
(iv) $\nabla \cdot \overline{F} \neq 0$

Q.2 Attempt Any Two

5

[10]

5

[10]

A

Α

4

5

1

(a) State modified Euler's theorem. If
$$u = \sin^{-1}\left(\frac{x^{\frac{1}{4}} + y^{\frac{1}{4}}}{x^{\frac{1}{6}} + y^{\frac{1}{6}}}\right)$$
, prove that

$$x^{2} \frac{\partial^{2} u}{\partial x^{2}} + 2xy \frac{\partial^{2} u}{\partial x \partial y} + y^{2} \frac{\partial^{2} u}{\partial y^{2}} = \frac{1}{144} \tan u [\tan^{2} u - 11].$$

- (b) Find the equations of the tangent plane and normal line at the point (-2, 1, -3) 5 A 2 to the ellipsoid $\frac{x^2}{4} + y^2 + \frac{z^2}{9} = 3$.
- (c) Find the extreme values of $x^3 + 3xy^2 15x^2 15y^2 + 72x$. 5 A 2

Q.3 Attempt Any Two

- (a) Eliminate the arbitrary function from the equation $z = xy + f(x^2 + y^2)$ 5 A 4
- (b) Solve:

i)
$$p^2 + q^2 = 1$$

ii) $p^2 - q^2 = x - y$

(c) Solve x(y-z)p + y(z-x)q = z(x-y) 5 A 4

Q.4 Attempt Any Two

- (a) Find the unit vector normal to the surface $x^2y + 2xz^2 = 8$ at(1,0,2). 5
- (b) Prove that the vector given below is both solenoidal and irrotational: $\bar{F} = (y^2 - z^2 + 3yz - 2x)\hat{i} + (3xz + 2xy)\hat{j} + (3xy - 2xz + 2z)\hat{k}$ (c) If $\bar{F} = 2xyz\hat{i} + (x^2z + 2y)\hat{j} + x^2y\hat{k}$, then 5 A 5
 - i. If \overline{F} is conservative, find its scalar potential ϕ
 - ii. Find the work done in moving a particle under this force field from (0,1,1) to (1,2,0).

R,A

2

SECTION B

			Marks	BL	CO
Q.5	Multiple-Choice Questions		[05]		
	(a) Let $A = \begin{bmatrix} 2 & 0 \\ 2 & -3 \end{bmatrix}$, then the Eigen value	ue of A are	1	U	1
	(i) 2,2	(ii) 2,-3			
	(iii) 1,1	(iv) -2,3			
	(b) The rank of the matrix $\begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & 0 \\ 1 & 1 & 1 \end{bmatrix}_{-1}^{-1}$		1	U	1
	(i) 0	(ii) 1			
	(iii) 2	(iv) 3			
	(c) The order and degree of the equation	$\left(\frac{dy}{dx}\right)^2 + 5y = x \text{ is } ___$	1	Α	3
	(i) 1,2	(ii) 1,6			
	(iii) 3,6	(iv) 2,1			
	(d) The solution of the equation $\frac{dy}{dx} + 3y$	=0 is	1	A	3
	(i) $y = 3e^{-x}$	(ii) $y = e^{-3x}$			
	(iii) $y = 3x^2$	(iv) $y = -\frac{3}{2}x^2$			
	(e) Particular integral of $(D^2 + 1)y = e^{-1}$	^x is	1	Α	3
	(i) $\frac{1}{2}e^{-x}$	(ii) $\frac{1}{2}xe^{-x}$			
	(iii) $-\frac{1}{2}e^{-x}$	$(\mathbf{iv}) -\frac{1}{2}xe^{-x}$			
Q.6	Attempt Any Two		[10]		
	(a) Find rank of the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 6 & 8 \\ 3 & 4 & 5 \end{bmatrix}$		5	Α	1
	(b) Find inverse of the matrix by Gauss J	Jordan method $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 3 \\ 1 & 0 & 8 \end{bmatrix}$	5	A	1

	(c) Solve the system of equations by Gauss elimination method $x - y + z = 3$	5	A	1
	2x - 3y + 5z = 10			
	x + y + 4z = 4			
Q.7	Attempt Any Two	[10]		
	(a) Check whether the given differential equation id exact or not	5	Α	3
	$(x^{4} - 2xy^{2} + y^{4})dx - (2x^{2}y - 4xy^{3} + \sin y)dy = 0$			
	Hence, find the general solution			
	(b) Solve $\frac{dy}{dx} + \frac{1}{x^2}y = 6e^{\frac{1}{x}}$	5	Α	3
	(c) Solve $\frac{dx}{dx} + \frac{y}{x} = x^3 y^3$	5	A	3
Q.8	Attempt Any Two	[10]		
	(a) Solve $(D^2 - 9)y = 0, y(0) = 2, y'(0) = -1$	5	Α	3
	(b) By using method of Variation of parameters, solve $y''+9y = \sec 3x$	5	A	3
	(c) Find the eigenvalues and eigenvectors of the matrix $\begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$.	5	A	1
