Enrolment No/Seat No.: ____________________

R.N.G.PATEL INSTITUTE OF TECHNOLOGY-RNGPIT

(An Autonomous College U/s UGC Act 1956)

B.TECH SEMESTER-I, SEMESTER END EXAMINATION – SUMMER 2025

Subject Code: 1SH102 **Date: 02-06-2025**

Subject Name: MATHEMATICS-I

Time: 11:00 AM to 02:00 PM **Total Marks: 70**

Instructions

1. It is **compulsory** for students to write **Enrolment No. /Seat No.** on the question paper.
2. Write answers of **Section A** and **Section B** in **separate answer books**.
3. Attempt all questions from both **Section A** and **Section B**.
4. Each section carries **35 marks**, with a total of **70 marks** for the examination.
5. The figures to the right of each question indicate full marks, make suitable assumptions with justification.
6. BL - Bloom's Taxonomy Levels (R-Remember, U-Understanding, A –Application, N –Analyze, E – Evaluate, C -Create), CO - Course Outcomes.

SECTION A

		Marks	BL	CO
Q.1	**Multiple-Choice Questions**	**[05]**		
	(a) If $u = \log\left(x^2 + y^2\right)$ then $\frac{\partial u}{\partial x} =$ ________	**1**	**A**	**2**
	(i) $\frac{2y}{x^2+y^2}$ **(ii)** $\frac{2}{x^2+y^2}$ **(iii)** $\frac{2x}{x^2+y^2}$ **(iv)** $\frac{xy}{x^2+y^2}$			
	(b) To find extreme values of $f(x,y)$, we solve:	**1**	**U**	**2**
	(i) $f_x = f_y = 0$ **(ii)** $f_x = f_y = 1$ **(iii)** $f_{xx} = f_{yy} = 0$ **(iv)** $f_{xy} = 0$			
	(c) The change of order of integration is useful when:	**1**	**R**	**4**
	(i) The integrand is discontinuous **(ii)** The inner integral is hard to evaluate **(iii)** The function is not continuous **(iv)** The limits are constants			
	(d) A vector field $\bar{F}$ is said to be a sink if___	**1**	**R**	**5**
	(i) $div\ \bar{F} = 0$ **(ii)** $div\ \bar{F} > 0$			

(iii) $div\,\bar{F} < 0$ (iv) None of the above

(e) The directional derivative of $f(x, y, z)$ in the direction of unit vector is given by: 1 R 5

(i) $\nabla f \cdot \hat{u}$ (ii) $\nabla f \times \hat{u}$

(iii) $\nabla \cdot \hat{u}$ (iv) $\nabla \cdot f$

Q.2 Attempt Any Two **[10]**

(a) If $u = \log(x^2 + y^2 + z^2)$, prove that $x\dfrac{\partial^2 u}{\partial y \partial z} = y\dfrac{\partial^2 u}{\partial z \partial x} = z\dfrac{\partial^2 u}{\partial x \partial y}$. **5 A 2**

(b) Find the equations of the tangent plane and normal line at the point $(-2,1,-3)$ to the ellipsoid $\dfrac{x^2}{4} + y^2 + \dfrac{z^2}{9} = 3$. **5 A 2**

(c) Find the extreme value of $x^3 + 3xy^2 - 3x^2 - 3y^2 + 4$ **5 A 2**

Q.3 Attempt Any Two **[10]**

(a) Evaluate $\iint (x^2 - y^2)dydx$ over the triangle with the vertices $(0,1)$, $(1,1)$ and $(1,2)$. **5 A 4**

(b) Change the order of integration and evaluate $\int_0^{\pi}\int_x^{\pi} \dfrac{\sin y}{y}\, dydx$ **5 A 4**

(c) Evaluate $\iint e^{y^2} dxdy$ over the region bounded by the triangle with vertices $(0,0),(2,1),(0,1)$ **5 A 4**

Q.4 Attempt Any Two **[10]**

(a) Show that $\bar{F} = (y+z)\hat{i} + (z+x)\hat{j} + (x+y)\hat{k}$, is irrotational and hence find its scalar potential function. **5 A 5**

(b) Find the work done, when a force $\bar{F} = (x^2 - y^2 + x)\hat{i} - (2xy + y)\hat{j}$ moves a particle from $(0,0)$ to the point $(1,1)$ along $y^2 = x$. **5 A 5**

(c) Prove that $\int_C \bar{F} \cdot d\bar{r} = 3\pi$, where $\bar{F} = z\hat{i} + x\hat{j} + y\hat{k}$ and C is the arc of the curve $\bar{r} = \cos t\hat{i} + \sin t\hat{j} + t\hat{k}$ from $t = 0$ to $t = 2\pi$. **5 A 5**

SECTION B

		Marks	BL	CO
Q.1	**Multiple-Choice Questions**	**[05]**		
	(a) For the matrix $A=\begin{bmatrix} 6 & 2 \\ 2 & 6 \end{bmatrix}$ the Eigen values are______. **(i)** 4, -8 **(ii)** 4, 8 **(iii)** -4, 8 **(iv)** -4, -8	**1**	**U**	**1**
	(b) The inverse of the matrix $\begin{bmatrix} 2 & 0 \\ 2 & 3 \end{bmatrix}$ is_____ . **(i)** $\begin{bmatrix} 3 & 0 \\ -2 & 2 \end{bmatrix}$ **(ii)** $\frac{1}{6}\begin{bmatrix} -3 & 0 \\ 2 & -2 \end{bmatrix}$ **(iii)** $\frac{1}{6}\begin{bmatrix} 3 & 0 \\ -2 & 2 \end{bmatrix}$ **(iv)** none of these	**1**	**U**	**1**
	(c) The order and degree of $\frac{d^3y}{dx^3}=\left[\left(\frac{dy}{dx}\right)^2+1\right]^{\frac{3}{2}}$ is ______. **(i)** 3, 2 **(ii)** 3, 1 **(iii)** 1, 2 **(iv)** 2, 3	**1**	**A**	**3**
	(d) The integrating factor of $\frac{dy}{dx}+y=e^{-x}$ is ______. **(i)** e^x **(ii)** e^{-x} **(iii)** $\log x$ **(iv)** x	**1**	**A**	**3**
	(e) The solution of $\left(D^2-5D+6\right)y=0$ is ______. **(i)** $y=c_1e^{2x}+c_2e^{-3x}$ **(ii)** $y=c_1e^{-2x}+c_2e^{-3x}$ **(iii)** $y=c_1e^{2x}+c_2e^{3x}$ **(iv)** $y=c_1e^{-2x}+c_2e^{3x}$	**1**	**A**	**3**
Q.2	**Attempt Any Two**	**[10]**		
	(a) Find rank of the matrix $\begin{bmatrix} 0 & 6 & 7 \\ -5 & 4 & 2 \\ 1 & -2 & 0 \end{bmatrix}$.	**5**	**A**	**1**

(b) Solve the system of equations by Gauss Jordan method $x+y+z=6$ $x+2y+3z=14$. $2x+4y+7z=30$	**5**	**A**	**1**
(c) Find Eigen value and Eigen vector $\begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}$.	**5**	**A**	**1**
Q.3 Attempt Any Two	**[10]**		
(a) Solve $\left(x^3+3xy^2\right)dx+\left(3x^2y+y^3\right)dy=0$.	**5**	**A**	**3**
(b) Solve $\dfrac{dy}{dx}+\dfrac{3y}{x}=\dfrac{\sin x}{x^3}$.	**5**	**A**	**3**
(c) Solve $\dfrac{dy}{dx}+\dfrac{y}{x}=\dfrac{y^2}{x^2}$.	**5**	**A**	**3**
Q.4 Attempt Any Two	**[10]**		
(d) Solve $\left(D^2-4D+4\right)y=e^{2x}+\cos 2x$.	**5**	**A**	**3**
(e) Using method of variation of parameter solve $\dfrac{d^2y}{dx^2}+9y=\sec 3x$.	**5**	**A**	**3**
(f) Using method of undetermined coefficients, solve the following equations $\left(D^2+4\right)y=8x^2$.	**5**	**A**	**3**

****