*Enrolment No/Seat No.:* ___________________

# R.N.G.PATEL INSTITUTE OF TECHNOLOGY-RNGPIT

***(An Autonomous College U/s UGC Act 1956)***

## B. Tech. SEMESTER-II, SEMESTER END EXAMINATION – SUMMER-2025

**Subject Code: 1SH202** **Date: 17-05-2025**

**Subject Name: NUMERICAL METHODS AND COMPLEX VARIABLES**

**Time: 11:00 AM to 02:00 PM** **Total Marks:70**

**Instructions**

1. It is **compulsory** for students to write **Enrolment No. /Seat No.** on the question paper.
2. Write answers of **Section A** and **Section B** in **separate answer books**.
3. Attempt all questions from both **Section A** and **Section B**.
4. Each section carries **35 marks**, with a total of **70 marks** for the examination.
5. The figures to the right of each question indicate full marks, make suitable assumptions with justification.
6. BL - Bloom's Taxonomy Levels (R-Remember, U-Understanding, A –Application, N –Analyze, E –Evaluate, C -Create), CO - Course Outcomes.

## SECTION A

**Marks BL CO**

**Q.1 Multiple-Choice Questions** **[05]**

**(a)** $\sinh z =$ ____ **1**

**(i)** $\frac{e^z - e^{-z}}{2}$ **(ii)** $\frac{e^z - e^{-z}}{2i}$

**(iii)** $\frac{e^z + e^{-z}}{2}$ **(iv)** $\frac{e^{iz} + e^{-iz}}{2i}$

**(b)** $\sqrt{-5+12i} =$ _____ **1**

**(i)** $\pm(3+2i)$ **(ii)** $\pm(2-3i)$

**(iii)** $\pm(2+3i)$ **(iv)** $\pm(3-2i)$

**(c)** If $z = \frac{-2}{1+\sqrt{3}i}$, $Arg(z) =$ ____ **1**

**(i)** $\frac{\pi}{3}$ **(ii)** $\frac{2\pi}{3}$

**(iii)** $-\frac{\pi}{3}$ **(iv)** $-\frac{\pi}{6}$

**(d)** In bracketing methods, the root of an equation is located between… **1**

**(i)** Two initial guesses without any condition **(ii)** Two points where the function has the same sign

**(iii)** Two points where the function has opposite signs **(iv)** The midpoints of two initial guesses

**(e)** Compared to the bisection method, the false position method generally… **1**

**(i)** converges slower

**(ii)** converges faster

**(iii)** is less stable

**(iv)** requires the second order derivative

**Q.2 Attempt Any Two** **[10]**

**(a)** Find a negative root of the equation $x^2-4x-10=0$ ,correct up to three decimal places by using bisection method. **5**

**(b)** Find a root between 0 and 1 of the equation $e^x \sin x = 1$, correct up to four decimal places by using Newton-Raphson method. **5**

**(c)** By using secant method, solve $xe^x - 1 = 0$, correct up to three decimal places, between 0 and 1. **5**

**Q.3 Attempt Any Two** **[10]**

**(a)** Considering $x$ as a dependent variable, fit a straight line to the following data. **5**

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| $x$ | 1 | 3 | 4 | 6 | 8 | 9 | 11 | 14 |
| $y$ | 1 | 2 | 4 | 4 | 5 | 7 | 8 | 9 |

**(b)** Fit a second-degree parabola $y = a + bx^2$ to the following data: **5**

| | | | | | |
|---|---|---|---|---|---|
| $x$ | 1 | 2 | 3 | 4 | 5 |
| $y$ | 1.8 | 5.1 | 8.9 | 14.1 | 19.8 |

**(c)** Fit a curve of the form $y = ax^b$ to the following data: **5**

| | | | | | |
|---|---|---|---|---|---|
| $x$ | 20 | 16 | 10 | 11 | 14 |
| $y$ | 22 | 41 | 120 | 89 | 56 |

**Q.4 Attempt Any Two** **[10]**

**(a)** State De Moivre's theorem. Evaluate $\dfrac{(\cos 2\theta + i\sin 2\theta)^{\frac{2}{3}}(\cos\theta - i\sin\theta)^2}{(\cos 3\theta - i\sin 3\theta)^2(\cos 5\theta - i\sin 5\theta)^{\frac{1}{3}}}$. **5**

**(b)** Find the product of all values of $\left(\dfrac{1}{2} + i\dfrac{\sqrt{3}}{2}\right)^{\frac{3}{4}}$. **5**

**(c)** Show that $f(z) = z|z|$ is not analytic anywhere. **5**

## SECTION B

| | | Marks | BL | CO |
|---|---|---|---|---|
| **Q.5** | **Multiple-Choice Questions** | **[05]** | | |
| | **(a)** Interpolation is a method of _____. | **1** | **1** | **2** |
| | **(i)** Interrelating    **(ii)** Estimating <br> **(iii)** Integrating    **(iv)** combining | | | |
| | **(b)** Newton's Forward interpolation formula can be used __________. | **1** | **1** | **2** |
| | **(i)** only for equally spaced intervals    **(ii)** only for unequally spaced intervals <br> **(iii)** for both equally and unequally spaced intervals    **(iv)** for unequally intervals | | | |
| | **(c)** Which method can be used for both equal and unequal intervals? | **1** | **1** | **2** |
| | **(i)** Lagrange's method    **(ii)** Divided difference method <br> **(iii)** Newton's method    **(iv)** both (i) and (ii) | | | |
| | **(d)** What method is commonly used to approximate solutions to ordinary differential equations by expanding the solution as a Taylor series? | **1** | **1** | **4** |
| | **(i)** Euler's Method    **(ii)** Modified Euler's Method <br> **(iii)** Taylor's Series Method    **(iv)** Runge-Kutta Method | | | |
| | **(e)** Which method utilizes a weighted average of slopes at different points within a step to achieve higher accuracy in numerical solutions of ordinary differential equations? | **1** | **1** | **4** |
| | **(i)** Euler's Method    **(ii)** Modified Euler's Method <br> **(iii)** Taylor's Series Method    **(iv)** Runge-Kutta Method | | | |
| **Q.6** | **Attempt Any Two** | **[10]** | | |
| | **(a)** Construct Newton's forward interpolation polynomial for the following data and hence find $y(5)$. | **5** | **3** | **2** |

| $x$ | 4 | 6 | 8 | 10 |
|---|---|---|---|---|
| $y$ | 1 | 3 | 8 | 16 |

| | | Marks | BL | CO |
|---|---|---|---|---|
| | **(b)** The population of a town is given below. Estimate the population for the year 1930 using suitable interpolation. | **5** | **3** | **2** |

| Year $x$ | 1891 | 1901 | 1911 | 1921 | 1931 |
|---|---|---|---|---|---|
| Population $y$ (in thousand) | 46 | 66 | 81 | 93 | 101 |

**(c)** By using Lagrange's formula, find $y$ when $x = 10$. **5** **3** **2**

| $x$ | 5 | 6 | 9 | 11 |
|---|---|---|---|---|
| $y$ | 12 | 13 | 14 | 16 |

**Q.7 Attempt Any Two** **[10]**

**(a)** Compute $f(-1)$ *and* $f(6)$ from the following values using Newton's divided difference formula: **5** **3** **2**

| $x$ | 1 | 2 | 4 | 7 |
|---|---|---|---|---|
| $f(x)$ | 10 | 15 | 67 | 430 |

**(b)** Evaluate $\int_0^1 e^x dx$ with $n=10$ using trapezoidal rule **5** **3** **4**

**(c)** Evaluate $\int_0^3 \frac{1}{1+x} dx$ taking $h = 0.5$ using Simpson's 3/8 rule **5** **3** **4**

**Q.8 Attempt Any Two** **[10]**

**(a)** Using the Taylor's series method, find correct to four decimal places, the value of y(0.1), given $\frac{dy}{dx} = y^2 + x$ and y(0)=1 **5** **3** **4**

**(b)** Using Euler's method find the approximate value of $y$ at x=1.5 taking h=0.1, given $\frac{dy}{dx} = \frac{y-x}{\sqrt{xy}}, \quad y(1) = 2$ **5** **3** **4**

**(c)** Obtain values of $y$ at $x=0.1$ using Runge-Kutta method of second order for the differential equation $\frac{dy}{dx} = x - y^2, y(0) = 1$ **5** **3** **4**

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