R.N.G.PATEL INSTITUTE OF TECHNOLOGY-RNGPIT (An Autonomous Institute U/s UGC Act 1956)

IM.Sc-IT SEMESTER-I, SEMESTER END EXAMINATION – SUMMER 2025

Subject Name: FUNDAMENTAL OF MATHEMATICS

Time: 11:00 AM to 02:00 PM Instructions 1. It is **compulsory** for students to write **Enrolment No. /Seat No.** on the question paper. 2. Write answers of Section A and Section B in separate answer books. 3. Attempt all questions from both Section A and Section B. 4. Each section carries **35 marks**, with a total of **70 marks** for the examination. 5. The figures to the right of each question indicate full marks, make suitable assumptions with justification. 6. BL - Bloom's Taxonomy Levels (R-Remember, U-Understanding, A – Application, N – Analyze, E – Evaluate, C -Create), CO - Course Outcomes. **SECTION A** Marks BL CO Q.1 **Multiple-Choice Questions** [05] (a) For a matrix A, if $AA^{\theta} = I$, then matrix A is called____ 1 U 3 (i)Symmetric (ii)Skew-symmetric (iii)Hermitian (iv)Unitary (b) A matrix A is said to be Hermitian if ... 1 U 3 (i) $(A^c)^T = A$ (ii) $(A^c)^T = -A$ $(\mathbf{iv})A^T = -A$ (iii) $A^T = A$ (c) In a Diagonalization of a matrix A, $D = P^{-1}AP$, the matrix P is known as: 1 U 3 (i) Symmetric Matrix (ii) Hermitian Matrix (iii) Modal Matrix (iv) Diagonal Matrix (d) Which of the following is known as second order partial derivative of f with U 5 1 respect to x(i) $\left(\frac{df}{dx}\right)^2$ (ii) $\frac{d^2f}{dx^2}$

(iv) $\left(\frac{\partial f}{\partial x}\right)^2$ (iii) $\frac{\partial^2 f}{\partial r^2}$

Date: 04-06-2025

Total Marks: 70

Subject Code: 1BS102

(e)	A function	f(x	is said to be continuous at	x = a if
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(i) $\lim_{x \to a} f(x)$ does not exist (ii) $\lim_{x \to a} f(x) = f(a)$ (iii) $\lim_{x \to a^{-}} f(x) \neq \lim_{x \to a^{+}} f(x)$ (iv) f(a) is not defined

Attempt Any Two Q.2 [10] (a) Find the characteristic equation of the matrix $\begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix}$ 5 A 3 (b) Verify Cayley Hamilton Theorem for the matrix $\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ (c) Find eigenvalues and eigen vectors of the matrix $\begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$ 5 3 Α 5 3 A **Attempt Any Two** Q.3 [10] (a) Let A be the matrix $\begin{bmatrix} 1 & 2 & -3 \\ 0 & 3 & 2 \\ 0 & 0 & -2 \end{bmatrix}$. Find eigen values of $A^3 + 5A^2 - 6A + 2I$. 5 A 3 (b) Check Continuity of the function: 5 Α 4 $f(x) = \begin{cases} x+2, & \text{if } x < 1\\ 1, & \text{if } x = 1\\ 2-x, & \text{if } x > 1 \end{cases}$ (c) Find derivative of the following: 5 Α 4 i. $\frac{d}{dx}[e^x \sin x]$ ii. $\frac{dx}{dx}[log(x^2 - 3x + 2)]$ **Attempt Any Two** 0.4 [10] (a) Verify Lagrange's Mean Value Theorem for $f(x) = x^3$ in [-2,2]. 5 U,A 5 (**b**) Find Maclaurin's series of e^x . 5 5 A (c) Find the extreme values of the function 5 A 5 $f(x, y) = x^2 + y^2 - 4x - 12y + 20$

1 U 4

SECTION B

			Marks	BL	CO
Q.5	Multiple-Choice Questions	[05]			
	(a) Set Operation Difference is		1	R	1
	(i) commutative	(ii) associative			
	(iii) both (i) and (ii)	(iv) neither (i) nor (ii)			
	(b) What is the value of $\begin{vmatrix} 1 & 0 \\ 4 & 2 \end{vmatrix}$.		1	Α	1
	(i) 2	(ii) 1			
	(iii) 0	(iv) not exist			
	(c) Which of the following represents the	e) Which of the following represents the empty set?			1
	(v) { }	(vi) Ø			
	$(\mathbf{vii}) \{ \emptyset \}$	(viii) both (i) and (ii)			
	(d) Which of the following is the correct diagram:	set notation for the following venn	1	U	1
	(i) $A \cup B \cup C$	$(\mathbf{ii}) \ A \cup (B \cap C)$			
	(iii) $B \cap C$	$(\mathbf{iv}) B \cup C$			
	(e) If the function $f: A \to B$ is then domain domain the function $f: A \to B$ is then domain	1	R	2	
	(i) A and B respectively	(ii) B and A respectively			
	(iii) Both domain and codomain are A	(iv) Both domain and codomain are B			
Q.6	Attempt Any Two		[10]		
	(a) Do as the following:		5	U,A	1
	(i) Represent the set $A = \{x \in \mathbb{N} \mid x$				
	(ii)Represent the set $A = \{2, 4, 6, 8, 1\}$				

	(b) Find domain, codomain and range of the following functions:	5	Α	1		
	(i) $f: A \to B$ defined by $f(x) = x^2 - 1$;					
	where $A = \{1, 2, 3, 4\}$ and $B = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$					
	(ii) If $f: \mathbb{Z} \to \mathbb{N}$ be a function defined by					
	f(x) = x + 3.					
	(c) Let the function $f: \mathbb{R} \to \mathbb{R}$ defined by $f(x) = x^2 + 1$ and $g: \mathbb{R} \to \mathbb{R}$ defined	5	Α	1		
	by $g(x) = 3x - 2$ then find $f \circ g$ and $g \circ f$.					
Q.7	Attempt Any Two	[10]				
	(a) Find inverse if the matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 3 \\ 1 & 0 & 8 \end{bmatrix}$ by using elementary	5	A	2		
	transformation.					
	(b) Reduce the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 7 \\ 3 & 1 & 2 \end{bmatrix}$ into upper triangular form (Echelon form).	5	A	2		
	(c) Solve the following system of equations using Cramer's Rule:	5	Α	2		
	5x - 7y + z = 11					
	6x - 8y - z = 15					
	3x + 2y - 6z = 7					
Q.8	Attempt Any Two	[10]				
	(a) Transform the matrix $\begin{bmatrix} 1 & 3 & 3 \\ 2 & 4 & 10 \\ 3 & 8 & 4 \end{bmatrix}$ into a unit matrix.	5	Α	2		
	(b) Reduce the matrix A to its normal form, when	5	Α	2		
	$A = \begin{bmatrix} 1 & 2 & -1 & 4 \\ 2 & 4 & 3 & 4 \\ 1 & 2 & 3 & 4 \\ -1 & -2 & 6 & -7 \end{bmatrix}.$					
	(c) Discuss the consistency of the following system of equations:	5	Α	2		
	x - y + z = 3					
	2x - 3y + 5z = 10					
	x + y + 4z = 4					
