R.N.G.PATEL INSTITUTE OF TECHNOLOGY-RNGPIT
(An Autonomous College U/s UGC Act 1956)

B. Tech. SEMESTER-I, SEMESTER END EXAMINATION - WINTER 2024

Subject Code: 1SH103	**Date: 09-12-2024**
Subject Name: CALCULUS	
Time: 11:00 AM to 02:00 PM	**Total Marks: 70**

Instructions

1. It is **compulsory** for students to write **Enrolment No. /Seat No.** on the question paper.
2. Write answers of **Section A** and **Section B** in **separate answer books**.
3. Attempt all questions from both **Section A** and **Section B**.
4. Each section carries **35 marks**, with a total of **70 marks** for the examination.
5. The figures to the right of each question indicate full marks, make suitable assumptions with justification.
6. BL - Bloom's Taxonomy Levels (R-Remember, U-Understanding, A –Application, N –Analyze, E – Evaluate, C -Create), CO - Course Outcomes.

SECTION A

		Marks	BL	CO

Q.1   Objective-Type Questions   **[05]**

(a) If $u = \log\left(x^2 + y^2\right)$ then $\dfrac{\partial u}{\partial x} =$ _____   **1   A   2**

    (i) $\dfrac{2y}{x^2 + y^2}$ **(ii)** $\dfrac{2}{x^2 + y^2}$

    (iii) $\dfrac{2x}{x^2 + y^2}$ **(iv)** $\dfrac{xy}{x^2 + y^2}$

(b) If $x^3 + y^3 + 3xy = 0$ then $\dfrac{dy}{dx} =$ _____   **1   A   2**

    (i) $-\dfrac{x^2 - y}{y^2 - x}$ **(ii)** $-\dfrac{x^2 + y}{y^2 + x}$

    (iii) $\dfrac{x^2 + y}{x - y^2}$ **(iv)** $\dfrac{x^2 + y}{x - y}$

(c) The partial differential equation formed by eliminating arbitrary constants from the equation $z = ax^2 + by^2$ is   **1   A   4**

    (i) $2z = xp + yq$ **(ii)** $z = xq + yp$

    (iii) $z = xp + yq$ **(iv)** none of these

(d) The solution of the equation $z = px + qy - pq$ is _____   **1   A   4**

    (i) $z = ax + by + ab$ **(ii)** $z = ax + by - pq$

    (iii) $z = ax + by - pq$ **(iv)** none of these

(e) If $\bar{r} = x\hat{i} + y\hat{j} + z\hat{k}$ then $div\,\bar{r}$ is_____   1   A   2

    (i) 0   **(ii)** 1

    (iii) 2   **(iv)** 3

Q.2   **Attempt Any Two**   **[10]**

    (a) If $u = f(x-y, y-z, z-x)$, then show that $\dfrac{\partial u}{\partial x} + \dfrac{\partial u}{\partial y} + \dfrac{\partial u}{\partial z} = 0$.   5   A   2

    (b) If $u = \cos ec^{-1}\left(\sqrt{\dfrac{x^{1/2} + y^{1/2}}{x^{1/3} + y^{1/3}}}\right)$ then prove that

$$x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y \frac{\partial^2 u}{\partial y^2} = \frac{1}{144}\tan u\left(13 + \tan^2 u\right)$$
   5   A   2

    (c) Find the extreme values of the function $x^3 + y^3 - 3x - 12y + 20$ 5   A   2

Q.3   **Attempt Any Two**   **[10]**

    (a) Eliminate the arbitrary function from the equation $z = xy + f(x^2 + y^2)$ 5   A   4

    (b) Solve $x^2 p + y^2 q = z^2$ 5   A   4

    (c) Solve (i) $\sqrt{p} + \sqrt{q} = 1$ (ii) $p - x^2 = q + y^2$ 5   A   4

Q.4   **Attempt Any Two**   **[10]**

    (a) Find the directional derivatives of the function $\phi = xyz$ at the point $(1,1,1)$ in the direction of the vector $\hat{i} + \hat{j} + \hat{k}$ 5   A   5

    (b) Prove that the vector $\bar{F} = (3x + 2y + 4z)\hat{i} + (2x + 5y + 4z)\hat{j} + (4x + 4y - 8z)\hat{k}$ is both solenoidal and irrotational   5   R,U   5

    (c) Find the work done, when a force $\bar{F} = (x^2 - y^2 + x)\hat{i} - (2xy + y)\hat{j}$ moves a particle from $(0,0)$ to the point $(1,1)$ along $y^2 = x$.   5   A   5

SECTION B

Q.5 **Objective-Type Questions**   **[05]**

(a) Let $A = \begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$, then the Eigen value of A are _____   **1   U   1**

    (i) $-6, -1$ (ii) $6, 1$

    (iii) $6, -1$ (iv) $-6, 1$

(b) The rank of the matrix is $\begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$ is_____   **1   U   1**

    (i) 0   (ii) 1

    (iii) 3   (iv) 2

(c) The solution of $\left(D^2 + 6D + 9\right) y = 0$ is _____   **1   A   3**

    (i) $\left(c_1 + c_2 x\right) e^{-3x}$ (ii) $c_1 e^{-3x}$

    (iii) $c_1 c_2 e^{-3x}$ (iv) $c_1 e^{-x}$

(d) The Wronskian of the two functions $\sin 2x$ and $\cos 2x$ is   **1   A   3**

    (i) 1   (ii) 2

    (iii) -1   (iv) -2

(e) Particular integral of $\left(D^2 + 4\right) y = \cos 2x$ is _____   **1   A   3**

    (i) $\dfrac{x \sin 2x}{2}$ (ii) $\dfrac{x \sin x}{4}$

    (iii) $\dfrac{x \sin 2x}{4}$ (iv) $x \sin 2x$

Q.6 **Attempt Any Two**   **[10]**

(a) Find inverse of the matrix by Gauss Jordan method $\begin{bmatrix} 2 & 3 & 4 \\ 4 & 3 & 1 \\ 1 & 2 & 4 \end{bmatrix}$ **5   A   1**

(b) Solve the system of equations by Gauss Elimination method $\begin{array}{l} x + y + 2z = 9 \\ 2x + 4y - 3z = 1 \\ 3x + 6y - 5z = 0 \end{array}$ **5   A   1**

(c) Find Eigen value and Eigen vector of the matrix $\begin{bmatrix} 4 & 6 & 6 \\ 1 & 3 & 2 \\ -1 & -4 & -3 \end{bmatrix}$ 5   A   1

Q.7 **Attempt Any Two**   **[10]**

(a) Form the differential equation by eliminating arbitrary constants from
$y = Ae^{-3x} + Be^{2x}$ 5   A   3

(b) Solve $\left(x^3 + 3xy^2\right)dx + \left(3x^2 y + y^3\right)dy = 0$ 5   A   3

(c) Solve $\dfrac{dy}{dx} + \dfrac{4x}{1+x^2} y = \dfrac{1}{\left(1+x^2\right)^3}$ 5   A   3

Q.8 **Attempt Any Two**   **[10]**

(a) Solve $\left(D^2 + 16\right)y = x^4 + e^{3x} + \cos 3x$ 5   A   3

(b) Using method of variation of parameter solve $\left(D^2 + 1\right)y = \cos ec\, x$ 5   A   3

(c) Using method of Undetermined coefficients, solve the following equations
$y'' + 4y = 8x^2$ 5   A   3

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