Enrolment No/Seat No.: ___________________

R.N.G.PATEL INSTITUTE OF TECHNOLOGY-RNGPIT

(An Autonomous College U/s UGC Act 1956)

B. Tech. SEMESTER-I, SEMESTER END EXAMINATION - WINTER 2024

Subject Code: 1SH102 **Date: 09-12-2024**

Subject Name: MATHEMATICS-I

Time: 11:00 AM to 02:00 PM **Total Marks: 70**

Instructions

1. It is **compulsory** for students to write **Enrolment No. /Seat No.** on the question paper.
2. Write answers of **Section A** and **Section B** in **separate answer books**.
3. Attempt all questions from both **Section A** and **Section B**;
4. Each section carries **35 marks**, with a total of **70 marks** for the examination.
5. The figures to the right of each question indicate full marks, make suitable assumptions with justification.
6. BL - Bloom's Taxonomy Levels (R-Remember, U-Understanding, A –Application, N –Analyze, E – Evaluate, C -Create), CO - Course Outcomes.

SECTION A

		Marks	BL	CO
Q.1	**Objective-Type Questions**	**[05]**		
	(a) The value of $\lim\limits_{\substack{x\to 0 \\ y\to 0}} \dfrac{2x^2+y}{4x^2-y}$ is	**1**	**A**	**2**
	(i) 1 **(ii)** 0 **(iii)** -1 **(iv)** does not exist			
	(b) If $u = x^2y + y^2z + z^2y$, then $\dfrac{\partial u}{\partial x}+\dfrac{\partial u}{\partial y}+\dfrac{\partial u}{\partial z} =$ _____	**1**	**A**	**2**
	(i) $2xy+2yz+2zx$ **(ii)** $(x+y+z)^2$ **(iii)** $\dfrac{1}{x+y+z}$ **(iv)** $x^2+y^2+z^2$			
	(c) $\int_{-1}^{1}\int_{0}^{2}(1-6x^2y)\,dx\,dy =$ _____	**1**	**A**	**4**
	(i) 1 **(ii)** 0 **(iii)** 4 **(iv)** 2			
	(d) If $\bar{F}$ is conservative, then _____	**1**	**U**	**5**
	(i) $\nabla\times\bar{F}=0$ **(ii)** $\nabla\times\bar{F}\neq 0$ **(iii)** $\nabla\bar{F}=0$ **(iv)** $\nabla\cdot\bar{F}=0$			

(e) If $\phi = xyz$, then the value of $\lvert grad\,\phi \rvert$ at $(1,2,-1)$ is **(i)** 0 **(ii)** 1 **(iii)** 2 **(iv)** 3		**1**	**A**	**5**
Q.2	**Attempt Any Two**	**[10]**		
(a) If $u = f(x-y, y-z, z-x)$, then show that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$.		**5**	**A**	**2**
(b) State modified Euler's theorem. If $u = \sin^{-1}\left(\frac{x^{1/4} + y^{1/4}}{x^{1/6} + y^{1/6}}\right)$, prove that $x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = \frac{1}{144} \tan u [\tan^2 u - 11]$.		**5**	**U,A**	**2**
(c) (i) Find the equations of the tangent plane and normal line to the surface $2x^2 + y^2 + 2z = 3$ at $(2,1,-3)$. (ii) If $x^y + y^x = c$, find $\frac{dy}{dx}$.		**5**	**A**	**2**
Q.3	**Attempt Any Two**	**[10]**		
(a) Change the order of integration and evaluate $\int_0^\infty \int_x^\infty \frac{e^{-y}}{y} dydx$.		**5**	**A**	**4**
(b) Evaluate $\iint \frac{x^2 y^2}{x^2 + y^2} dxdy$ over the region bounded by the circles $x^2 + y^2 = a^2$ and $x^2 + y^2 = b^2$ $(a > b)$.		**5**	**A**	**4**
(c) Evaluate $\iint (x^2 - y^2) dydx$ over the triangle with the vertices $(0,1)$, $(1,1)$ and $(1,2)$.		**5**	**A**	**4**
Q.4	**Attempt Any Two**	**[10]**		
(a) (i) Find the directional derivative of $\phi = xy^2 + yz^2$ at point $(2,-1,1)$ in the direction of the vector $\hat{i} + 2\hat{j} + 2\hat{k}$. (ii) If $\bar{A} = (ax^2 y + yz)\hat{i} + (xy^2 - xz^2)\hat{j} + (2xyz - 2x^2 y^2)\hat{k}$ is solenoidal, find the constant a.		**5**	**R,U**	**5**
(b) A vector field is given $\bar{F} = (x^2 - y^2 - x)\hat{i} - (2xy + y)\hat{j}$. Show that the field is irrotational and find its scalar potential.		**5**	**A**	**5**
(c) Prove that $\int_C \bar{F} \cdot d\bar{r} = 3\pi$, where $\bar{F} = z\hat{i} + x\hat{j} + y\hat{k}$ and C is the arc of the curve $\bar{r} = \cos t\hat{i} + \sin t\hat{j} + t\hat{k}$ from $t = 0$ to $t = 2\pi$.		**5**	**A**	**5**

SECTION B

			Marks	BL	CO
Q.5	**Objective-Type Questions**		**[05]**		

(a) The Eigen value of $A\begin{bmatrix}1 & 1 & 1\\ 0 & 2 & 1\\ 0 & 0 & 3\end{bmatrix}$ are ______ **1 U 1**

(i) 0,0,0 **(ii)** 1,1,1

(iii) 1,2,3 **(iv)** -1,-2,-3

(b) The rank of the matrix is $\begin{bmatrix}1 & 1 & 1\\ -1 & -1 & -1\\ 1 & 1 & 1\end{bmatrix}$ is_____ **1 U 1**

(i) 1 **(ii)** 0

(iii) 3 **(iv)** 2

(c) The equation $M(x,y)dx + N(x,y)dy = 0$ is exact if ______ **1 A 3**

(i) $\frac{\partial M}{\partial x} = \frac{\partial N}{\partial y}$ **(ii)** $\frac{\partial M}{\partial x} = -\frac{\partial N}{\partial y}$

(iii) $\frac{\partial M}{\partial y} = -\frac{\partial N}{\partial x}$ **(iv)** $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

(d) The solution of $y'' - 4y' + 4y = 0$ is ______ **1 A 3**

(i) $y = (c_1 + c_2 x)e^{2x}$ **(ii)** $y = (c_1 + c_2 x)e^{-2x}$

(iii) $y = c_1 e^{2x} + c_2 e^{-2x}$ **(iv)** $y = c_1 \cos 2x + c_2 \sin 2x$

(e) The particular integral of $y'' + y = e^{-x}$ is ______ **1 A 3**

(i) $\frac{1}{2}xe^{-x}$ **(ii)** $\frac{1}{2}e^{-x}$

(iii) $-\frac{1}{2}xe^{-x}$ **(iv)** $-\frac{1}{2}e^{-x}$

Q.6 **Attempt Any Two** **[10]**

(a) Find inverse of the matrix by Gauss Jordan method $\begin{bmatrix}0 & 1 & 2\\ 1 & 2 & 3\\ 3 & 1 & 1\end{bmatrix}$ **5 A 1**

(b) Investigate for what values of λ and μ the equations $\begin{array}{l} x+2y+z=8 \\ 2x+2y+2z=13 \\ 3x+4y+\lambda z=\mu \end{array}$ have (i) no solution (ii) a unique solution, and (iii) many solutions | **5** | **A** | **1**

(c) Find Eigen value and Eigen vector $\begin{bmatrix} 1 & 2 & 2 \\ 0 & 2 & 1 \\ -1 & 2 & 2 \end{bmatrix}$ | **5** | **A** | **1**

Q.7 Attempt Any Two **[10]**

(a) Solve $\left(y^2-x^2\right)dx+2xydy=0$ | **5** | **A** | **3**

(b) Solve $\frac{dy}{dx}+y\tan x=\sin 2x, \quad y(0)=0$ | **5** | **A** | **3**

(c) Solve $\frac{dy}{dx}+\frac{y}{x}=x^3y^3$ | **5** | **A** | **3**

Q.8 Attempt Any Two **[10]**

(a) Solve (i) $\left(D^3+1\right)y=0$ (ii) $\left(D^3-3D^2-D+3\right)y=0$ | **5** | **A** | **3**

(b) Solve $y'''-3y''+3y'-y=4e^x$ | **5** | **A** | **3**

(c) Using method of variation of parameter solve $\frac{d^2y}{dx^2}+y=\cos ecx$ | **5** | **A** | **3**

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