Enrolment No/Seat No.: _____

R.N.G.PATEL INSTITUTE OF TECHNOLOGY-RNGPIT

(An Autonomous College U/s UGC Act 1956)

IMSc-IT. SEMESTER-I, SEMESTER END EXAMINATION - WINTER 2024

Subject Code: 1BS102 **Date: 13-12-2024**

Subject Name: FUNDAMENTAL OF MATHEMATICS

Time: 11:00 AM to 02:00 PM **Total Marks: 70**

Instructions

1. It is **compulsory** for students to write **Enrolment No. /Seat No.** on the question paper.
2. Write answers of **Section A** and **Section B** in **separate answer books**.
3. Attempt all questions from both **Section A** and **Section B**.
4. Each section carries **35 marks**, with a total of **70 marks** for the examination.
5. The figures to the right of each question indicate full marks, make suitable assumptions with justification.
6. BL - Bloom's Taxonomy Levels (R-Remember, U-Understanding, A –Application, N –Analyze, E – Evaluate, C -Create), CO - Course Outcomes.

SECTION A

		Marks	BL	CO
Q.1	**Objective-Type Questions**	**[05]**		
(a)	The characteristic equation of a matrix A is obtained by ___	**1**	**R**	**3**
	(i) $\|A - \lambda I\| = 0$ **(ii)** $\|A+\lambda I\| = 0$			
	(iii) $[A - \lambda I] = 0$ **(iv)** $\|A\lambda I\| = 0$			
(b)	Which of the following matrices satisfies the Cayley-Hamilton theorem?	**1**	**U**	**3**
	(i) Any diagonalizable matrix. **(ii)** Any invertible matrix.			
	(iii) Any square matrix. **(iv)** Only symmetric matrices.			
(c)	The derivative of a function *f* is given by:	**1**	**R**	**4**
	(i) $f'(x) = \lim_{h\to 0} \frac{f(x+h)+f(x)}{h}$ **(ii)** $f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$			
	(iii) $f'(x) = \lim_{x\to 0} \frac{f(x+h)+f(x)}{x}$ **(iv)** $f'(x) = \lim_{x\to 0} \frac{f(x+h)-f(x)}{x}$			
(d)	The symbol $\frac{\partial}{\partial x}$ represents:	**1**	**R**	**5**
	(i) The derivative with respect to *x* in a multivariable function. **(ii)** The second derivative of a function.			
	(iii) The integration with respect to *x* **(iv)** The derivative with respect to all variables			

(e) Which of the following indeterminate forms can be resolved using L' Hospital's Rule? **1 R 5**

(i) 0^0 **(ii)** $\frac{0}{0}$

(iii) $\infty - \infty$ **(iv)** ∞

Q.2 Attempt Any Two **[10]**

(a) Find the eigen values and the corresponding eigen vectors of the matrix $\begin{bmatrix} -2 & 5 & 4 \\ 5 & 7 & 5 \\ 4 & 5 & -2 \end{bmatrix}$. **5 A 3**

(b) Show that the given matrix $\begin{bmatrix} -i & 3+2i & -2-i \\ -3+2i & 0 & 3+4i \\ 2-i & -3-4i & -2i \end{bmatrix}$ is Skew Hermitian. **5 A 3**

(c) Let $A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$. Find matrix P such that $P^{-1}AP$ is diagonalizable matrix. **5 A 3**

Q.3 Attempt Any Two **[10]**

(a) Evaluate the following: **5 A 4**

(i) $\lim_{x\to 0} \frac{\cos x - 1}{x}$

(ii) $\lim_{x\to \infty} \frac{2x^2-3x+4}{x^2+4x-1}$

(b) Verify whether the function $f(x) = \begin{cases} x+2, & if\ x < 1 \\ 1\ , & if\ x = 1 \\ 2-x, & if\ x > 1 \end{cases}$ **5 A 4**

(c) Find derivative of the following: **5 A 4**

(i) $\frac{d}{dx}(e^x \sin x)$

(ii) $\frac{d}{dx}[\log(x^2 - 3x + 2)]$

Q.4 Attempt Any Two **[10]**

(a) State Rolle's Theorem and verify it for $y = x^2 + 1$ in$[-1,1]$. **5 R,A 5**

(b) Expand the function $f(x) = 2x - x^2$ in terms of $(x + 3)$ by using Taylor's series. **5 A 5**

(c) Find the extreme values of the function **5 A 5**

$$f(x, y) = x^2 + y^2 - 6x - 8y + 9$$

SECTION B

		Marks	BL	CO
Q.5	**Objective-Type Questions**	**[05]**		
(a)	Power set of Empty set has exactly ____ element/elements.	**1**	**U**	**1**
	(i) 0 **(ii)** 1			
	(iii) 2 **(iv)** ∞			
(b)	A Set which described with the help of a statement is called ___	**1**	**R**	**1**
	(i) Tabular form **(ii)** Roster form			
	(iii) Set-Builder form **(iv)** Both (i) and (ii)			
(c)	Which of the following represents the empty set?	**1**	**U**	**1**
	(i) { } **(ii)** $\emptyset$			
	(iii) $\{\emptyset\}$ **(iv)** Both (i) and (ii)			
(d)	Which of the following is the set builder form of a set $A = \{1, 2, 4, 8, 16, 32 \ldots\}$?	**1**	**U**	**1**
	(i) $\{x/x \in \mathbb{N}\}$ **(ii)** $\{2x/x \in W\}$			
	(iii) $\{x^2/x \in \mathbb{N}\}$ **(iv)** $\{2^x/x \in W\}$			
(e)	If the function $f: A \to B$ is injective and surjective then _____	**1**	**U**	**2**
	(i) f^{-1} exists **(ii)** f is invertible			
	(iii) Both (i) and (ii) **(iv)** Neither (i) and (ii)			
Q.6	**Attempt Any Two**	**[10]**		
(a)	Check whether the function $f: \mathbb{R} \to \mathbb{R}$ defined by $f(x) = 2x - 1$ is bijective or not.	**5**	**U**	**1**
(b)	Find domain, co-domain and range of the following functions:	**5**	**A**	**1**

(i) $f: A \to B$ defined by $f(x) = 3x - 4$;
where $A = \{1,2,3,4\}$ and
$B = \{-2, -1, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

(ii) If $f: \mathbb{Z} \to \mathbb{N}$ be a function defined by
$f(x) = x^2 + 3$.

(c) Let the function $f: \mathbb{R} \to \mathbb{R}$ defined by $f(x) = x^2$ and $g: \mathbb{R} \to \mathbb{R}$ defined by $g(x) = 2x + 1$ then find $f \circ g$ and $g \circ f$. **5 A 1**

Q.7 Attempt Any Two **[10]**

(a) Find inverse if the matrix $A = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$ by using elementary transformation. **5 A 2**

(b) Reduce the matrix $\begin{bmatrix} 1 & 2 & 3 & 2 \\ 1 & 2 & 3 & 4 \\ 2 & 6 & 7 & 5 \end{bmatrix}$ into normal form. **5 A 2**

(c) Solve the following system of equations using Cramer's Rule: **5 A 2**

$$5x - 6y + 4z = 15$$
$$7x + 4y - 3z = 19$$
$$2x + y + 6z = 46$$

Q.8 Attempt Any Two **[10]**

(a) Transform the matrix $\begin{bmatrix} 1 & 3 & 3 \\ 2 & 4 & 10 \\ 3 & 8 & 4 \end{bmatrix}$ into a unit matrix. **5 A 2**

(b) Find rank of the matrix $\begin{bmatrix} 1 & 2 & -1 & 3 \\ 4 & 1 & 2 & 1 \\ 3 & -1 & 1 & 2 \\ 1 & 2 & 0 & 1 \end{bmatrix}$. **5 A 2**

(c) Discuss the consistency of the following system of equations: **5 A 2**

$$2x + 3y + 4z = 11$$
$$x + 5y + 7z = 15$$
$$3x + 11y + 13z = 25.$$

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