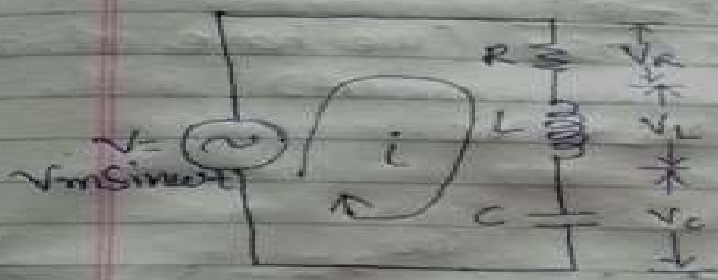
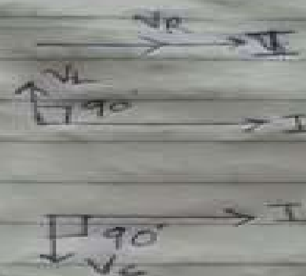
CHAPTER-2

AC CIRCUITS

R-L-C Series Circuit :-

Mr. K. C. Shah

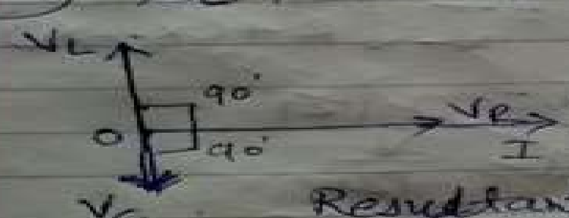




→ In R-L-C Series circuit there are three possibilities.

1) $X_L > X_C$ ⟶ Behaved as a R-Inductive ckt.
2) $X_L < X_C$ ⟶ Behaved as a R-C ckt.
3) $X_L = X_C$. ⟶ Behaved as Resistive circuit.

1) $X_L > X_C$ → So $V_L > V_C$.



Resultant Voltage $V = \sqrt{V_R^2 + (V_L - V_C)^2}$

$$= \sqrt{(IR)^2 + (I(X_L - X_C))^2}$$

$$= I\sqrt{R^2 + (X_L - X_C)^2}$$

$$V = I\,|Z|$$

$$\therefore I = \frac{V}{|Z|}$$

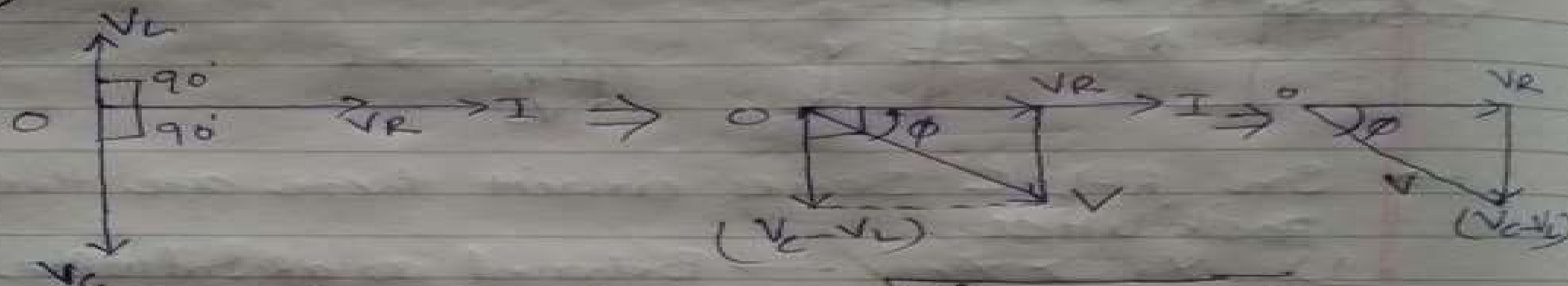
$$i = \frac{V\angle 0}{Z\angle\phi}$$

$\therefore \; i = I_m \angle -\phi$

$\therefore \; \boxed{i = I_m \sin(\omega t - \phi)}$

and Power Avg $P_{av} = VI\cos\phi$ (We have already derived in R-L series circuit).

2) $X_L < X_C \longrightarrow$ So, $V_L < V_C$.



Resultant Voltage $V = \sqrt{V_R^2 + (V_C - V_L)^2}$

$= \sqrt{(IR)^2 + (I(X_C - X_L))^2}$

$= I\sqrt{R^2 + (X_C - X_L)^2}$

$V = I|Z|$

$\therefore \; I = \dfrac{V}{|Z|}$

$\therefore \; i = \dfrac{V\angle 0}{Z\angle -\phi} = I_m \angle +\phi$

$\boxed{i = I_m \sin(\omega t + \phi)}$

and Power Avg. $P_{av} = VI\cos\phi$ (We have already derived in R-C series ckt.)

3) $X_L = X_C \rightarrow$ So, $V_L = V_C$, So, R-L-C series circuit behaves as Resistive circuit and it is known as R-L-C resonance circuit.

★ **Resonance in R-L-C Series Circuit :-**

→ In Resonance R-L-C series circuit

$X_L = X_C$.

$\therefore V_L = V_C$.

Resultant Voltage $V = \sqrt{V_R^2} = V_R$.

$V = IZ$

$$\boxed{\therefore Z = \frac{V}{I} = R}$$

→ So, In Resonance R-L-C series circuit the Impedance is purely resistive and minimum. Here the Current is maximum and such circuit is known as Resonance circuit and frequency is known as Resonance frequency.

→ Resonance circuit is also known as Acceptor Circuit. It is used as tunning circuit in Radio.

→ In Resonance circuit $V_L = V_C$, so both voltage neutralize each other leaving behind a resistive voltage drop in circuit which to be equal to the Supply Voltage.

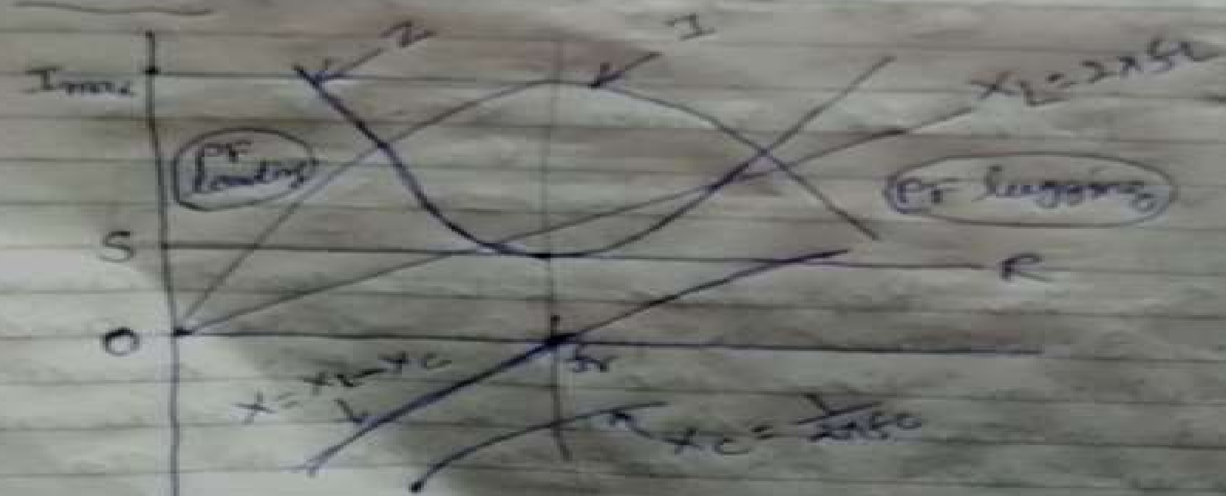
→ In Resonance $X_L = X_C$

$$2\pi f_r L = \frac{1}{2\pi f_r C}$$

$$\therefore f_r^2 = \frac{1}{(2\pi)^2 LC}$$

Resonant frequency → $$\boxed{\therefore f_r = \frac{1}{2\pi\sqrt{LC}}}$$

* Graphical Representation of RLC Series Resonance Circuit:



→ The behaviour of R-L-C Series circuit in respect of Resonance as following.

1) As the frequency is changed, the value of Resistance R is independent of frequency. It doesn't vary with frequency as shown the horizontal line SR as shown in the Graph.

2) X_L is directly proportional to the frequency and represented by $O-X_L$ in the graph.

3) X_C is inversely proportional to the frequency and shown in the graph.

4) Net reactive reactance $X = X_L - X_C$ is also shown and it is passed through the resonant frequency f_r.

5) Impedance Z is minimum and it is equal to R at resonant frequency and exponential to both the side as shown in the graph.

6) Current I is maximum at resonant frequency f_r and decreased on both the side of f_r.

7) When $f < f_r$, the power factor is leading because it behaves as capacitive circuit. & when $f > f_r$, the power factor is lagging because it behaves as inductive circuit.

Q-factor (Quality factor):-

→ At Resonance Z = R and therefore the current is maximum in the circuit and will cause large voltage drop across L and C and $V_L = V_C$ and opposition to the voltage drop across L and C. So, voltage drop occured in Resistor R which is megnified the voltage. This voltage magnification is called quality factor (Q-factor).

→ So, Quallity factor (Q-factor) is defined as the ratio of Potential drop across inductor or capacitor and the Potential drop across the resistor.

$$\therefore \text{Q-factor} = \frac{\text{Potential drop across Inductor/Capacitor}}{\text{Potential drop across Resistor}}$$

$$= \frac{\cancel{I} X_L}{\cancel{I} R}$$

$$= \frac{2\pi f_r L}{R} = \frac{\omega_r L}{R} \quad \left(f_r = \frac{1}{2\pi\sqrt{LC}} \right)$$

$$\text{Q-factor} = \frac{\cancel{2\pi} L}{R \times \cancel{2\pi} \times \sqrt{LC}} = \frac{1}{R} \times \frac{L}{\sqrt{LC}}$$

$$\boxed{\therefore \text{Q-factor} = \frac{1}{R}\sqrt{\frac{L}{C}}}$$

→ Q-factor of a series circuit Indicates how many time the potential drop L and C is greater than apply voltage.

e.g. $V = 250V \rightarrow \text{Q-factor} = 20 \rightarrow V_L = V_C = 5000V$

Mr. Kevin C. Shah

1) A.C. Parallel Circuit :-

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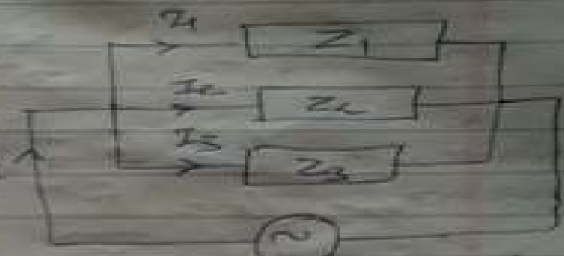


$$I = I_1 + I_2 + I_3 \quad \text{--- (1)}$$

$$\frac{V}{Z} = \frac{V}{Z_1} + \frac{V}{Z_2} + \frac{V}{Z_3}$$

$$\therefore \frac{1}{Z} = \frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3}$$

$$\therefore Y = Y_1 + Y_2 + Y_3 \quad \text{--- (2)}$$

Here Y = admittance. (Unit is Siemens).

Eq. (1) can write

$$VY = VY_1 + VY_2 + VY_3.$$

$$\therefore Y = Y_1 + Y_2 + Y_3 \quad \text{--- (3)}$$

Eq (2) & (3) both are same.

Conductance :- (G)

$$Z = R \pm jX$$

So, $Y = \frac{1}{R \pm jX}$

$$\therefore Y = \frac{R \mp jX}{R \pm jX} \times \frac{1}{(R \mp jX)}$$

$$= \frac{(R \mp jX)}{(R^2 + X^2)}$$

$$Y = \frac{R}{R^2+X^2} + \frac{jX}{R^2+X^2}$$

$$\therefore Y = \frac{R}{Z^2} + \frac{jX}{Z^2}$$

$$\therefore Y = G \mp jB \Rightarrow |Y| = \sqrt{G^2+B^2}, \ \phi = \tan^{-1}\left(\frac{B}{G}\right)$$

Here $G = \frac{R}{Z^2}$ = Conductance (Unit is Siemens)

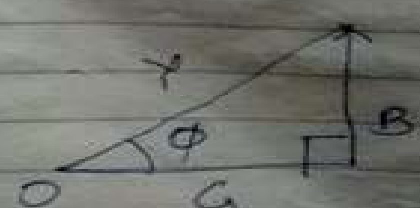
Susceptance (B) :-

$B = \frac{X}{Z^2}$ = Susceptance (Unit is Siemens)

Polarity	+	—
Reactance, X	Inductive	Capacitive
Susceptance, B	Capacitive	Inductive

* Admittance Triangle :-

(−B Inductive Susceptance)



(+B Capacitive Susceptance)

* Types of Parallel AC Circuits:-

1) Resistance in parallel with pure Inductor (R-L parallel)
2) Resistance in parallel with pure capacitor (R-C parallel)
3) Series combination of R and L in parallel with a capacitor in parallel (RL series & C in parallel).

→ 1) Resistance in parallel with Pure Inductor (R-L parallel)

$V = V_m \sin\omega t$

Phasor diagram:-

$$I = I_R + I_L$$

$$I = \frac{V}{R} + \frac{V}{X_L}$$

Impedance:- Here $Z_1 = R + j0 = R\angle 0^\circ\ \Omega$

$Z_2 = 0 + jX_L = X_L\angle 90^\circ\ \Omega$

$$Z = Z_1 \parallel Z_2$$

$$= \frac{Z_1 \times Z_2}{Z_1 + Z_2}$$

$$= \frac{R\angle 0^\circ \times X_L\angle 90^\circ}{(R + j0 + 0 + jX_L)}$$

$$= \frac{R X_L \angle 90^\circ}{R + jX_L}$$

$$\therefore \; Z = \frac{R X_L \angle 90^\circ}{\sqrt{R^2 + X_L^2} \; \angle \tan^{-1}\left(\frac{X_L}{R}\right)}$$

$$\boxed{\therefore \; Z = \frac{R X_L}{|Z_s|} \angle (90^\circ - \phi_s)}$$

where $|Z_s| = \sqrt{R^2 + X_L^2}$ & $\phi = \angle \tan^{-1}\left(\frac{X_L}{R}\right)$

Voltage & Current Waveforms :-

Here $V = V_m \sin \omega t$

& we get $i = I_m \sin(\omega t - \phi)$.

Power factor :-

P. F. $= \cos\phi$

Here Power factor is lagging by angle ϕ. Means Current is lagging by angle ϕ to the voltage.

→ 2) Resistance in Parallel with Pure Capacitor (RC parallel)

$V = V_m \sin \omega t$

Phasor diagram :-

$$I = I_R + I_C$$

$$I = \frac{V}{R} + \frac{V}{X_C}$$

Impedance :- Here $Z_1 = R + j0 = R\angle 0^\circ \ \Omega$

$$Z_2 = 0 - jX_C = X_C \angle -90^\circ \ \Omega$$

$$\therefore Z = \frac{Z_1 Z_2}{Z_1 + Z_2}$$

$$= \frac{R\angle 0^\circ \times X_C \angle -90^\circ}{R + j0 + 0 - jX_C}$$

$$= \frac{R X_C \angle -90^\circ}{R - jX_C}$$

$$Z = \frac{R X_C \angle -90^\circ}{\sqrt{R^2 + X_C^2} \ \angle \tan^{-1}\left(\frac{-X_C}{R}\right)}$$

$$\therefore Z = \frac{R X_C}{|Z_S|} \angle (90^\circ + \phi_S).$$

where $|Z_S| = \sqrt{R^2 + X_C^2}$ & $\phi_S = \angle -\tan^{-1}\left(\frac{X_C}{R}\right)$

Voltage & Current Waveform :-

Here $V = V_m \sin \omega t$

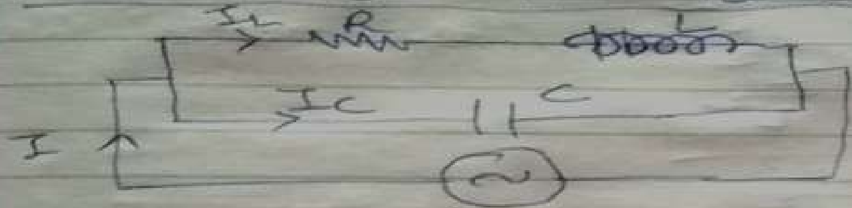
& we get $i = I_m \sin(\omega t + \phi)$.

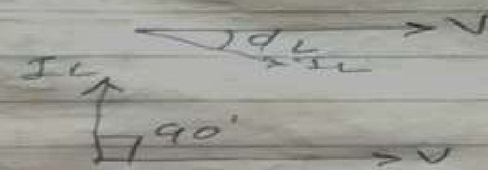
Power factor :-

$$P.F. = \cos\phi$$

Here P.F. is leading because I leads voltage by an angle of ϕ.

Resonance in Parallel Circuit:-





Phasor diagram:-

For Resonance, $I_C = I_L \sin\phi_L$.

$$I_C = \frac{V}{X_C}, \quad I_L = \frac{V}{Z_L}.$$

$$Z_L = R + jX_L = \sqrt{R^2 + X_L^2}$$

$$\phi_L = \tan^{-1}\left(\frac{X_L}{R}\right)$$

$$\sin\phi_L = \frac{X_L}{Z_L}.$$

$$\therefore \frac{V}{X_C} = \frac{V}{Z_L} \times \frac{X_L}{Z_L}.$$

$$\therefore Z_L^2 = X_L \cdot X_C.$$

$$\therefore R^2 + (\omega_r L)^2 = \omega_r L \times \frac{1}{\omega_r C}.$$

$$\therefore (\omega_r L)^2 = \frac{L}{C} - R^2$$

$$\therefore \omega_r^2 L^2 = \frac{L}{C} - R^2.$$

$$\therefore \omega_r^2 = \frac{1}{LC} - \frac{R^2}{L^2}$$

$$\therefore \omega_r = \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

$$\therefore 2\pi f_r = \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}.$$

$$\boxed{\therefore f_r = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}.} \leftarrow \text{Resonant frequency}$$

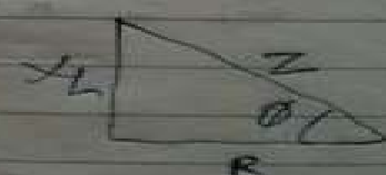
If R of the coil is negligible, so, $\frac{R^2}{L^2} \to 0$.

$$\therefore \boxed{f_r = \frac{1}{2\pi}\sqrt{\frac{1}{LC}} = \frac{1}{2\pi\sqrt{LC}}}$$

→ To find the Dynamic Impedance,

$$I = I_L \cos\phi$$

$$\frac{V}{Z} = \frac{V}{Z_L} \cdot \frac{R}{Z_L}$$



$$\therefore \frac{1}{Z} = \frac{R}{Z_L^2}.$$

But $Z_L^2 = X_L \cdot X_C = \frac{L}{C}$.

$$\therefore \frac{1}{Z} = \frac{RC}{L}$$

$$\boxed{\therefore Z_D = \frac{L}{RC}}$$

$\boxed{\text{Parallel Resonance Ckt. is a Rejector Circuit}}$

resultant Current $I = \frac{V}{Z} = \frac{V}{4RC}$

→ In parallel resonance circuit I will be minimum because Impedance is maximum.

$\boxed{I = \frac{V}{Z} \text{ will be minimum}}$

$Q\text{-}\cancel{\text{factor}} = \frac{I_C}{I} = \frac{I_L \sin\phi_L}{I_L \cos\phi_L} = \tan\phi_L$

$= \frac{X_L}{R} = \frac{2\pi f_r L}{R}$

* Comparison between Series & Parallel Resonance:

Sr. no.	Description	Series Resonance	Parallel Resonance
1	f_r	$\frac{1}{2\pi\sqrt{LC}}$	$\frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$
2	Impedance Z	Minimum	Maximum
3.	Z_r (Effective Impedance)	R	$\frac{L}{CR}$
4	Current	$\frac{V}{R}$	$\frac{V}{4RC}$
5.	Power factor	unity	unity
6.	Magnifies	Voltage	Current.
7.	Q-factor	$\frac{\omega_r L}{R}$	$\frac{\omega_r L}{R}$
8	Application (used as)	Acceptor	Rejector.