CHAPTER-2

AC CIRCUITS

3-PHASE CIRCUITS

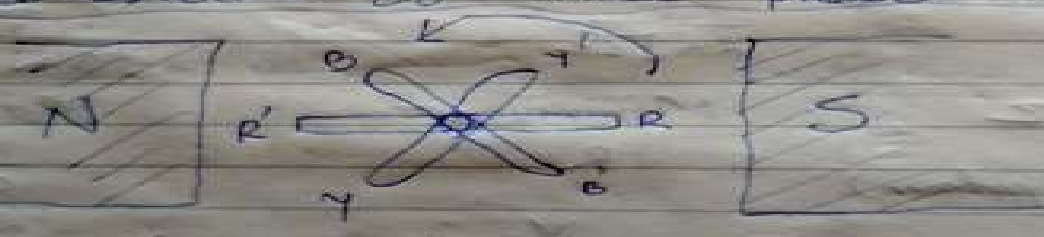
Three Phase Balanced System

→ Polyphase - No. of phases are available.

→ Advantages:

1) More output
2) Smaller size.
3) 3-phase motors are self starting
4) More power is transmitted.
5) Smaller cross-sectional area of conductors
6) Better power factor.
7) Horsepower rating of motors and ~~too~~ KVA of transformers are higher.

→ Generation of Three phase EMF:

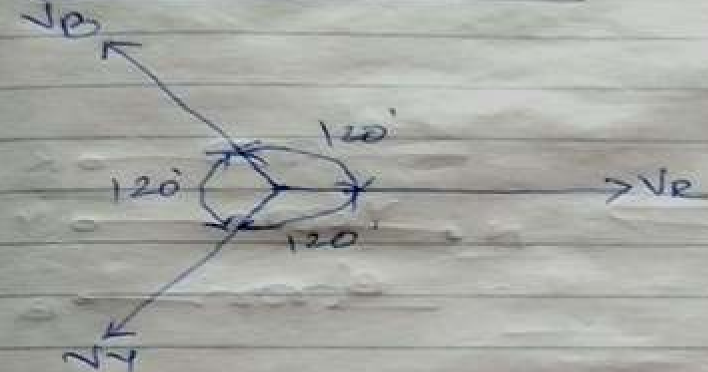


$V_R = V_m \sin \omega t$

$V_Y = V_m \sin(\omega t - 120°)$

$V_B = V_m \sin(\omega t - 240°) = V_m \sin(\omega t + 120°)$

Phasor representation



Vector addition of three phase voltages at any instant is zero.

$\Rightarrow V_R + V_Y + V_B = 0.$

LHS $V_R + V_Y + V_B$

$= V_m \sin\omega t + V_m \sin(\omega t - 120°) + V_m \sin(\omega t + 120°)$

$= (V_m + j0) + [V_m \cos 120° - j V_m \sin 120°] + [V_m \cos 120° + j V_m \sin 120°]$

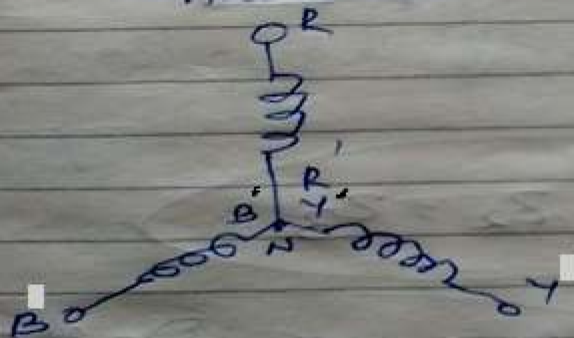
$= V_m - \frac{V_m}{2} - \frac{V_m}{2}$

$= V_m - V_m$

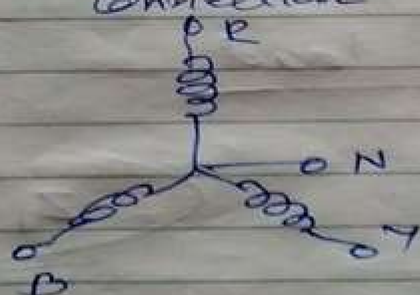
$\boxed{V_R + V_Y + V_B = 0}$

Types of Three phase Supply Connections.

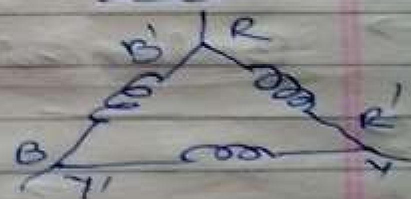
Three phase three wire (Wye) connection



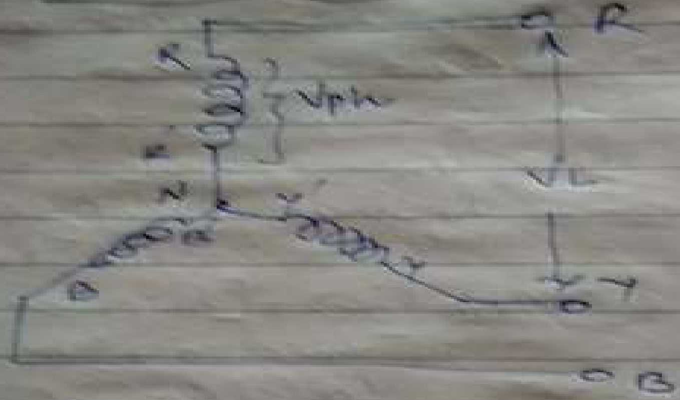
Three phase four wire star connection



Three phase three wire delta connection:



→ **Star Connection:-**



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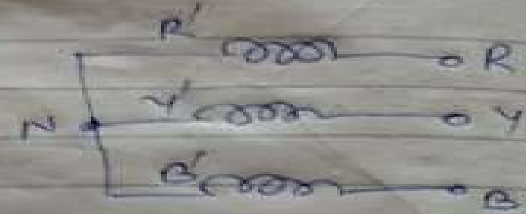

→ **Delta Connection:-**

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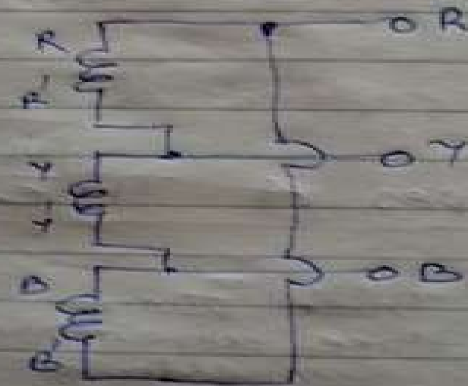
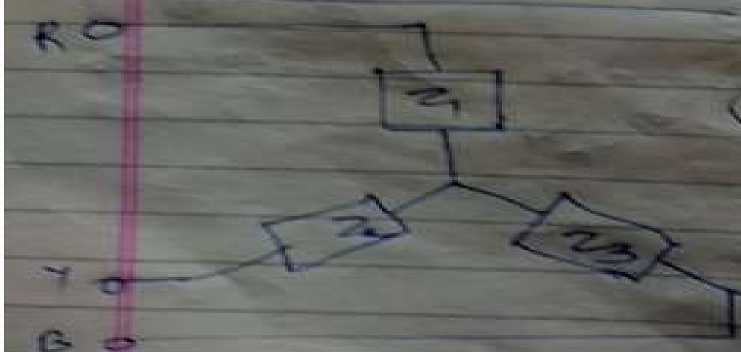


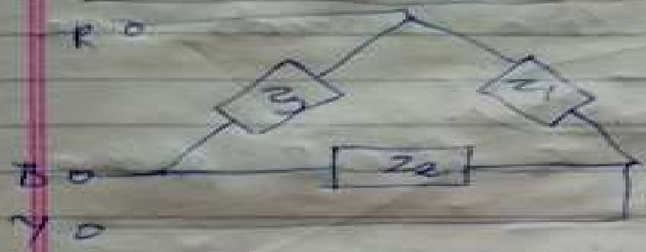
→ **Balanced Load**



$Z_1 = Z_2 = Z_3$

Star or Delta

Unbalanced load :-



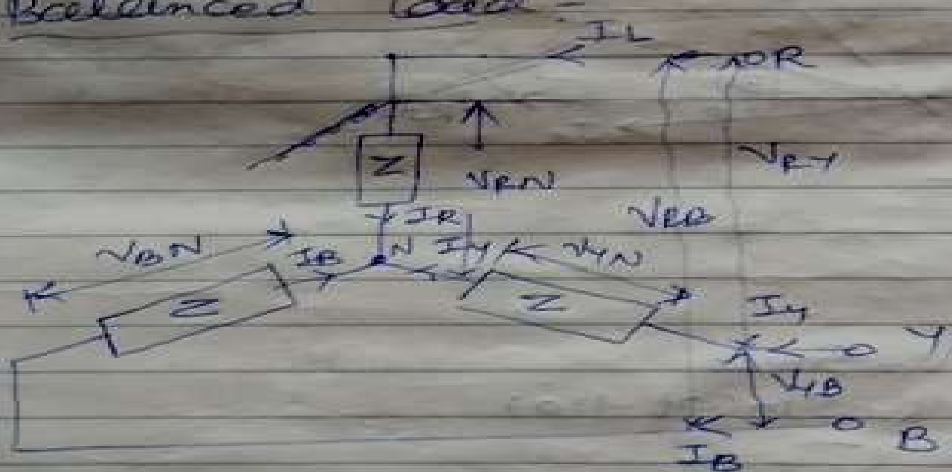
$\boxed{Z_1 \neq Z_2 \neq Z_3}$

Star or Delta

1) Phase Voltage → V_{ph} → Voltage across ~~between~~ phase (winding)
2) Phase Current → I_{ph} → Current through phase (winding)
3) Line Voltage → V_L → Voltage across line (phases).
4) Line Current → I_L → Current through line (phase).

✱ Voltage & Current Relations in Star Connection

Balanced load :-



Here In 3-phase balanced load star Connection

Phase Voltages – V_{ph} → V_{RN}, V_{YN}, V_{BN}

Line Voltages – V_L → V_{RY}, V_{YB}, V_{BR}.

Phase Currents → I_{ph} → I_R, I_Y, I_B.

Line Currents – I_L → I_R, I_Y, I_B.

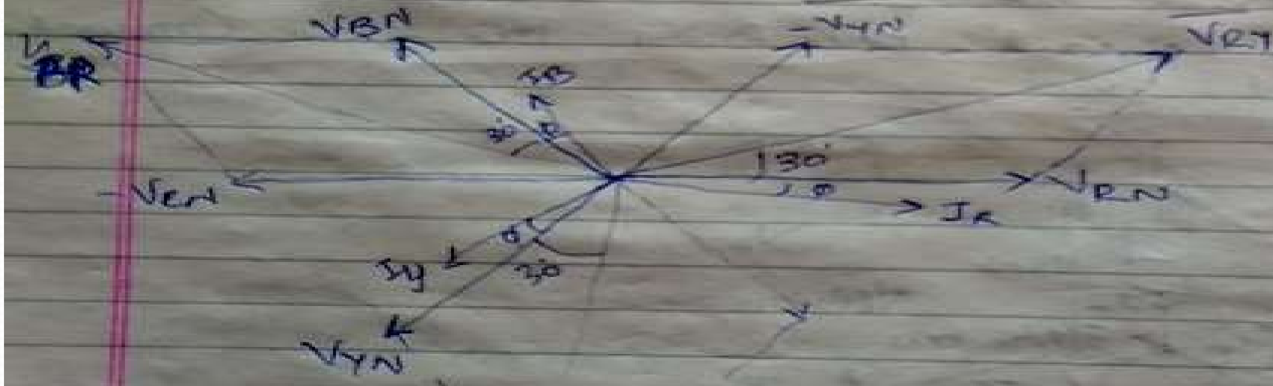
So, In 3-phase Star connections with Balanced Load, Phase Current & Line Current are equal. So, $I_L = I_{ph}$.

Now, for getting Relations of Line & phase Voltage,

Now, $\overline{V_{RY}} = \overline{V_{RN}} + \overline{V_{NY}}$

But $\overline{V_{NY}} = -\overline{V_{YN}}$

$\therefore \overline{V_{RY}} = \overline{V_{RN}} - \overline{V_{YN}}$



Let $V_R = V_m \sin\omega t$

$V_Y = V_m \sin(\omega t - 120°)$

$V_{RY} = \overline{V_R} - \overline{V_Y}$

$= V_m\cos 0° + jV_m \sin 0 - (V_m\cos 120° + jV_m\sin 120°)$

$= V_m + j0 - (-0.5V_m - j0.866V_m)$

$\overline{V_{RY}} = V_m + 0.5V_m + j0.866V_m$

$$\therefore \; V_{RY} = \sqrt{(1.5V_m)^2 + (0.866V_m)^2} \; \angle \tan^{-1}\left(\frac{0.866}{1.5}\right)$$

$$= \sqrt{3}\, V_m \; \angle 30^\circ$$

$$\boxed{V_L = \sqrt{3}\, V_{ph}} \quad \angle 30^\circ$$

Total Power $P_T = 3\,P_{ph}$

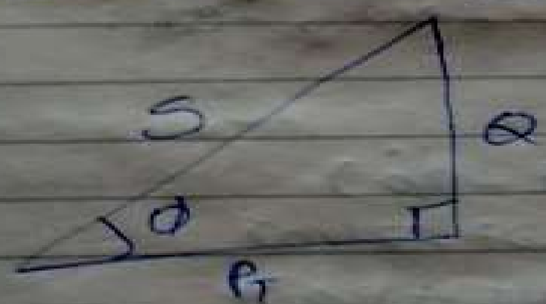
$$= 3 \times V_{ph} \times I_{ph} \cos\phi$$

$$= 3 \times \frac{1}{\sqrt{3}} V_L \times I_L \times \cos\phi$$

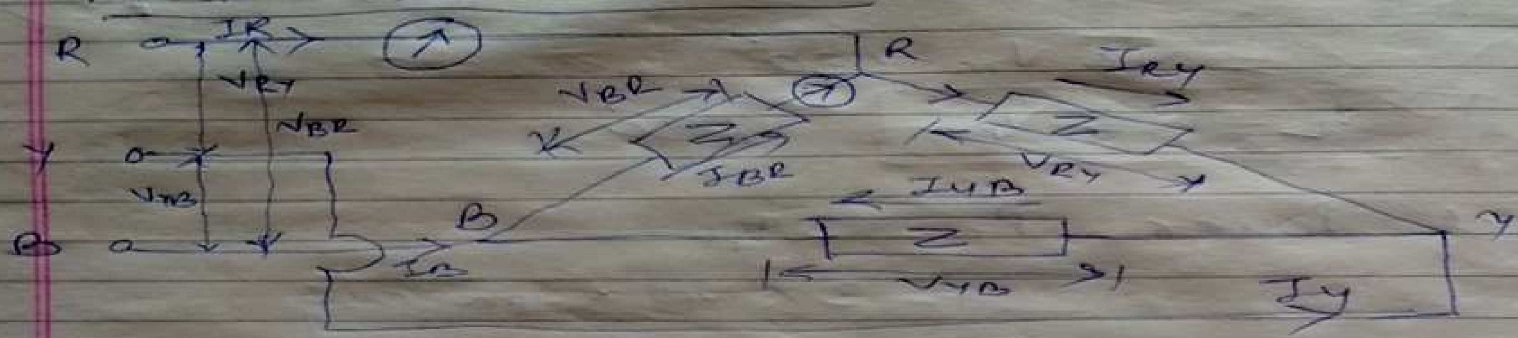
$$\boxed{P_T = \sqrt{3}\, V_L I_L \cos\phi}$$

Apparent Power $(S) = \sqrt{3}\, V_L I_L$.

Reactive Power $(Q) = \sqrt{3}\, V_L I_L \sin\phi$.



* Voltage & Current Relation in Balanced Load delta Connection :-



Here Line Voltage $V_L = V_{RY}, V_{BR}, V_{YB}$

Phase Voltage $V_{ph} = V_{RY}, V_{YB}, V_{BR}$.

So, $V_L = V_{ph}$

Line Current $= I_R, I_Y, I_B$.

phase Current $= I_{RY}, I_{YB}, I_{BR}$.

Here $I_{RY} = I_m \sin \omega t$

$I_{YB} = I_m \sin(\omega t - 120^\circ)$

$I_{BR} = I_m \sin(\omega t - 240)$

$= I_m \sin(\omega t + 120^\circ)$

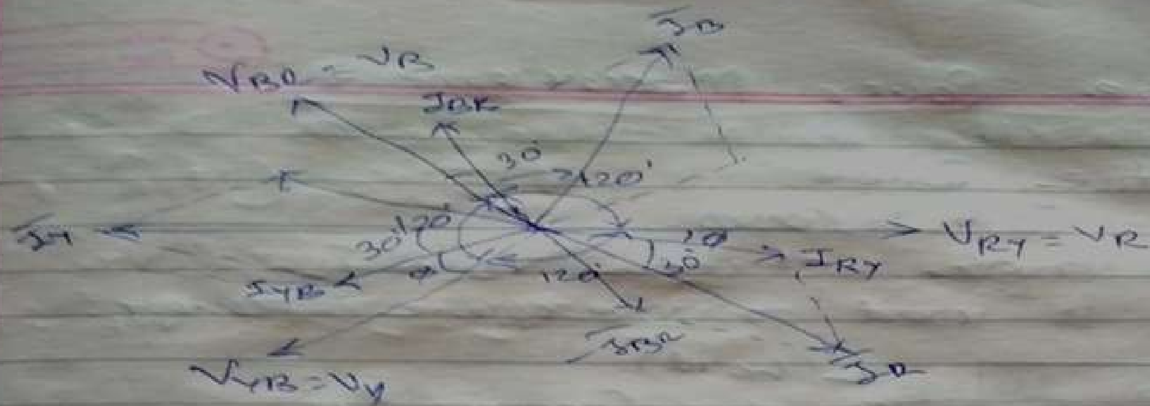
Applying KCL,

$\bar{I}_Y + \bar{I}_{RY} = \bar{I}_{YB}$

$\bar{I}_R + \bar{I}_{BR} = \bar{I}_{RY}$

$\bar{I}_B + \bar{I}_{YB} = \bar{I}_{BR}$

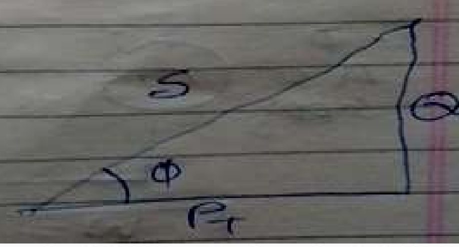
$\therefore \bar{I}_R = \bar{I}_{RY} - \bar{I}_{BR}$



$$\therefore I_R = I_m \sin\omega t - I_m \sin(\omega t + 120^\circ)$$
$$= I_m \cos 0 + j I_m \sin 0 - I_m \cos 120^\circ - j I_m \sin 120^\circ$$
$$= I_m + 0 + 0.5 I_m - j0.866 I_m$$
$$I_R = 1.5 I_m - j0.866 I_m$$
$$\therefore I_R = \sqrt{(1.5 I_m)^2 + (-0.866 I_m)^2} \angle \tan^{-1}\left(\frac{-0.866}{1.5}\right)$$
$$= \sqrt{3} I_m \angle -30^\circ$$
$$I_R = \sqrt{3} I_m$$
$$\boxed{I_L = \sqrt{3} I_{ph} \angle -30^\circ}$$

$$P_T = 3 V_{ph} I_{ph} \cos\phi$$
$$= 3 \times V_L \times \frac{I_L}{\sqrt{3}} \cos\phi$$
$$\boxed{P_T = \sqrt{3} V_L I_L \cos\phi}$$

$$S = V_{ph} I_{ph} = \sqrt{3} V_L I_L$$
$$Q = V_{ph} I_{ph} \sin\phi = \sqrt{3} V_L I_L \sin\phi$$



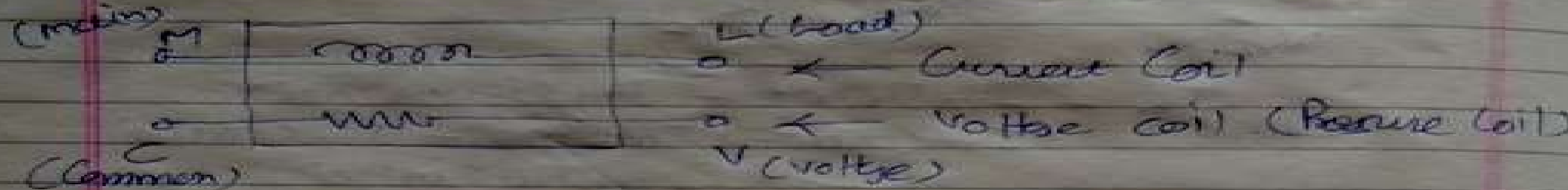
* Three Phase Power Measurement :-

Total active Power

$$P_T = \sqrt{3}\, V_L I_L \cos\phi$$

ϕ is the angle between V_L & I_L

Wattmeter is used to measure power



→ Current coil is between M and L it is connected in series. Resistance is small

→ Voltage Coil (Pressure Coil) is between C and V. It is connected across the supply to measure voltage. Resistance is large

$$\underbrace{W}_{\text{Wattmeter reading}} = \underbrace{V_{pc}}_{\text{voltage across voltage coil}} \times \underbrace{I_c}_{\text{Current through Current coil}} \times \cos(\underbrace{V_{pc} \hat{} I_c}_{\text{Angle between } V_{pc} \text{ and } I_c})$$

→ Single phase Power measurement using a Wattmeter

$$\boxed{W = V_{ph} I_{ph} \cos\phi}$$

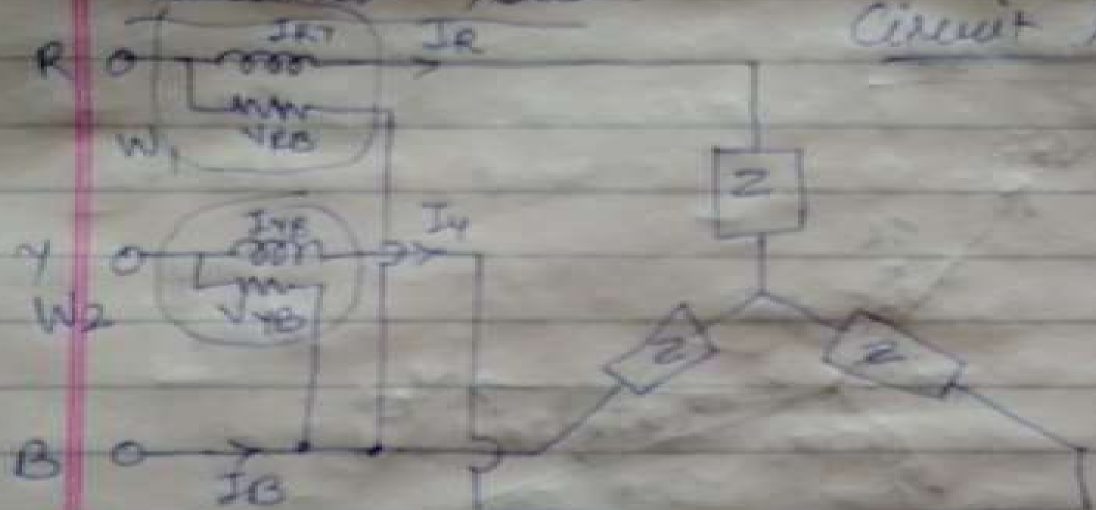
* Power Measurement in 3-ϕ AC Circuits:-

There are main three methods.

1) One Wattmeter method
2) Two Wattmeter method
3) Three Wattmeter method

* Two Wattmeter method for Star Connection Balanced Load:

Circuit Diagram:-



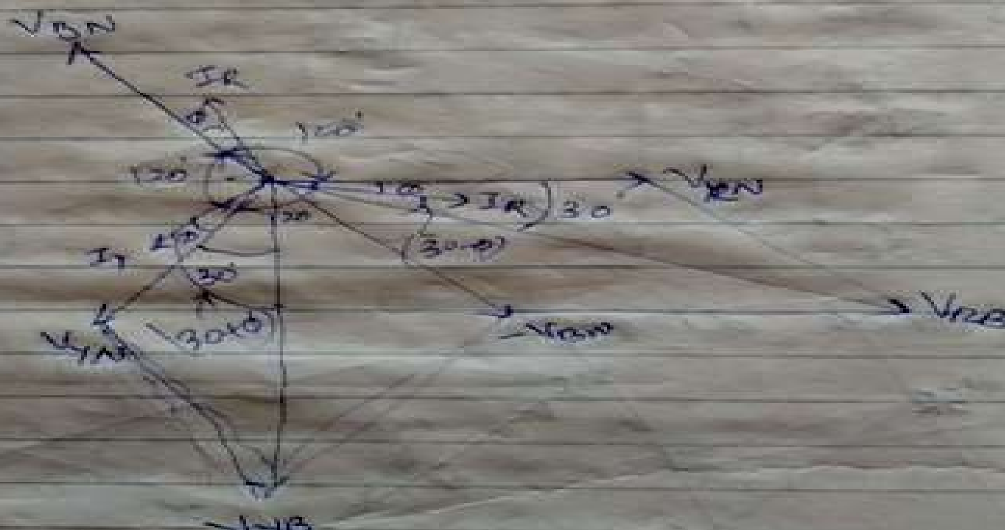
Now from Wattmeter $W_1 = V_{RB} I_R \cos(V_{RB} \hat{} I_R)$

$= V_L I_L \cos(V_{RB} \hat{} I_R)$

& for Wattmeter $W_2 = V_{YB} I_Y \cos(V_{YB} \hat{} I_Y)$

$W_2 = V_L I_L \cos(V_{YB} \hat{} I_Y)$

Phasor Diagram



from the phasor diagram

$W_1 = V_L I_L \cos(30 - \phi)$

$W_2 = V_L I_L \cos(30 + \phi)$.

Now, total active power

$W = W_1 + W_2$

$= V_L I_L \cos(30-\phi) + V_L I_L \cos(30+\phi)$

$= V_L I_L [\cos(30-\phi) + \cos(30+\phi)]$

$= V_L I_L [\cos 30 \cos\phi + \sin 30 \sin\phi + \cos 30 \cos\phi - \sin 30 \sin\phi]$

$$= V_L I_L \left[2\cos 30 \cos\phi\right]$$

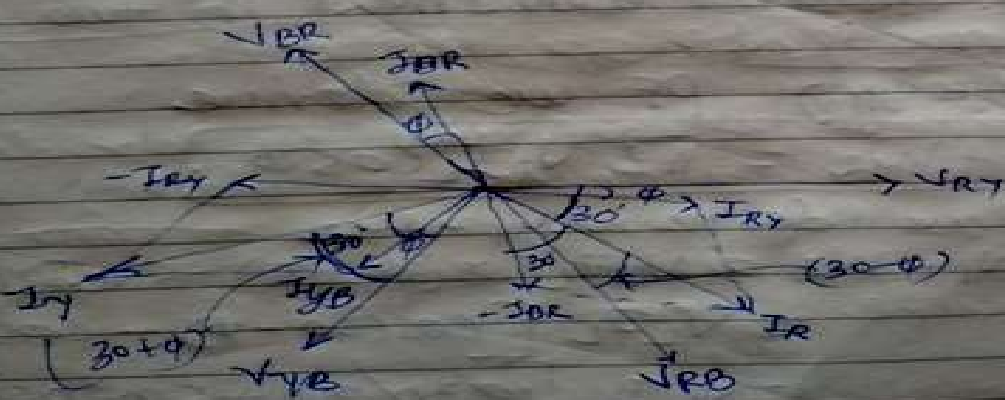
$$= V_L I_L \left[2 \times \frac{\sqrt{3}}{2} \times \cos\phi\right]$$

$$W = \sqrt{3} V_L I_L \cos\phi$$

$$\boxed{\therefore \quad W = W_1 + W_2 = \sqrt{3} V_L I_L \cos\phi}$$

* Two Wattmeter method for Delta Connection Balanced Load =

for Wattmeter $W_1 = V_{RB} I_R \cos(V_{RY} \hat{} I_R)$
Wattmeter $W_2 = V_{YB} I_Y \cos(V_{YB} \hat{} I_Y)$.



from phasor diagram

$W_1 = V_{RB} I_R \cos(30-\phi)$

$W_1 = V_L I_L \cos(30-\phi)$

& $W_2 = V_{YB} I_Y \cos(30+\phi)$

$W_2 = V_L I_L \cos(30+\phi)$

So, total Active Power

$$W = W_1 + W_2 = V_L I_L \cos(30-\phi) + V_L I_L \cos(30+\phi)$$

$$= V_L I_L [\cos(30-\phi) + \cos(30+\phi)]$$

$$= V_L I_L [\cos 30 \cos\phi + \cancel{\sin 30 \sin\phi} + \cos 30 \cos\phi - \cancel{\sin 30 \sin\phi}]$$

$$= V_L I_L [2 \cos 30 \cos\phi]$$

$$= V_L I_L \left[2 \times \frac{\sqrt{3}}{2} \times \cos\phi\right]$$

$$\boxed{W = W_1 + W_2 = \sqrt{3} V_L I_L \cos\phi}$$

* Power factor Measurement by two wattmeter ~~meter~~ method:

For lagging PF we derive

$$\boxed{W_1 + W_2 = \sqrt{3} V_L I_L \cos\phi}$$

Now, we derive $W_1 - W_2$

$$\therefore W_1 - W_2 = V_L I_L \cos(30-\phi) - V_L I_L \cos(30+\phi)$$

$$= V_L I_L [\cos(30-\phi) - \cos(30+\phi)]$$

$$= V_L I_L [\cancel{\cos 30 \cos\phi} + \sin 30 \sin\phi - \cancel{\cos 30 \cos\phi} + \sin 30 \sin\phi]$$

$$= V_L I_L \left[2 \sin 30 \sin\phi\right]$$
$$= V_L I_L \left[2 \times \frac{1}{2} \times \sin\phi\right]$$
$$\boxed{W_1 - W_2 = V_L I_L \sin\phi}$$

Now
$$\frac{W_1 - W_2}{W_1 + W_2} = \frac{\cancel{V_L I_L} \sin\phi}{\sqrt{3}\,\cancel{V_L I_L} \cos\phi}$$
$$\frac{W_1 - W_2}{W_1 + W_2} = \frac{1}{\sqrt{3}} \tan\phi$$
$$\therefore \tan\phi = \sqrt{3}\left[\frac{(W_1 - W_2)}{(W_1 + W_2)}\right]$$
$$\therefore \phi = \tan^{-1}\left[\frac{\sqrt{3}(W_1 - W_2)}{(W_1 + W_2)}\right]$$
$$\therefore \text{Power factor } \cos\phi = \cos\left[\tan^{-1}\left(\frac{\sqrt{3}(W_1 - W_2)}{(W_1 + W_2)}\right)\right]$$

if $\phi = 0$, $\quad W_1 = V_L I_L \cos 30 = \frac{\sqrt{3}}{2} V_L I_L$

$\quad W_2 = V_L I_L \cos 30 = \frac{\sqrt{3}}{2} V_L I_L$

$$\boxed{\therefore W = 2W_1 = 2W_2}$$

if $\phi = 60^\circ$, $\quad W_1 = V_L I_L \cos(-30^\circ) = \frac{\sqrt{3}}{2} V_L I_L$

$\quad W_2 = V_L I_L \cos 90^\circ = 0$.

$$\therefore \boxed{W = W_1}$$

if $\phi = 90^\circ$ $\quad W_1 = V_L I_L \cos(-60^\circ) = \frac{1}{2} V_L I_L$

$\quad W_2 = V_L I_L \cos(120^\circ) = \frac{1}{2} V_L I_L$

$$\boxed{\therefore W = W_1 = W_2}$$

Reactive Power $Q = \sqrt{3} V_L I_L \sin\phi$

$$\boxed{Q = \sqrt{3}(W_1 - W_2)}$$

* Merits (Advantages) of two wattmeter method:-

1) We can used it for the balanced as well as unbalanced load.
2) For star connection, No neutral point is Required
3) The Delta load need not be open to connect the wattmeter
4) For balanced load it is possible to measure the power factor ~~rating~~ along with Power.
5) We need to use only two wattmeters to measure power in three phase system.
6) It is possible to measure the reactive power for the balanced load.

* Demerits (Disadvantages):-

1) The Polarity of W_1 and W_2 depends on the power factor so unless we identify them correctly they can lead us to wrong result
2) This method is not used for three phase four wire system.