PRESENTATION ON
Time Response Analysis of system

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- Standard Test Signals
- What is time response ?
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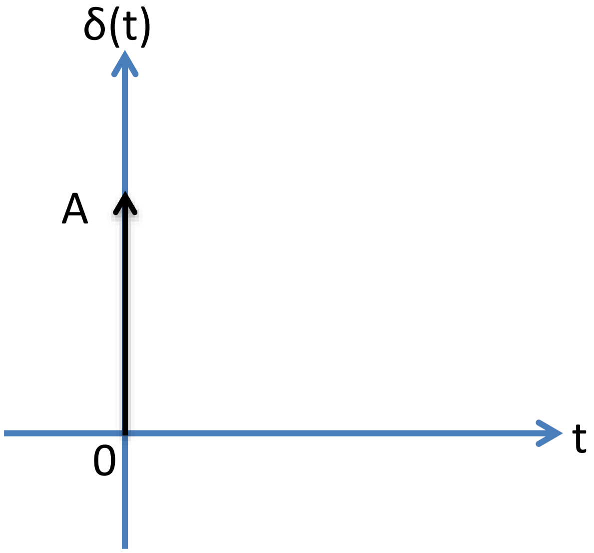
- Analysis of First order system
- Analysis of Second order system

Standard Test Signals

- Impulse signal
  - The impulse signal imitate the sudden shock characteristic of actual input signal.

$$\delta(t) = \begin{cases} A & t = 0 \\ 0 & t \neq 0 \end{cases}$$

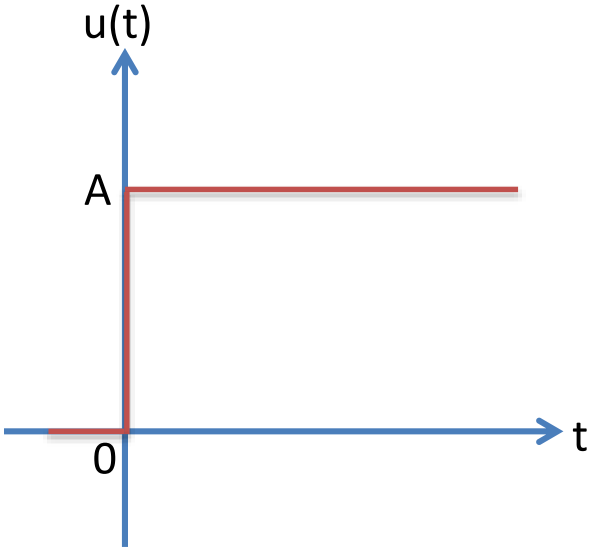


  - If A=1, the impulse signal is called unit impulse signal.

Standard Test Signals

- Step signal
  - The step signal imitate the sudden change characteristic of actual input signal.

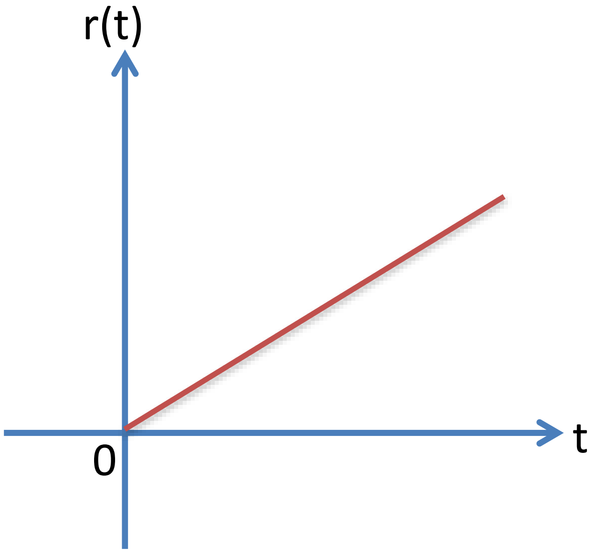
$$u(t) = \begin{cases} A & t \geq 0 \\ 0 & t < 0 \end{cases}$$



  - If A=1, the step signal is called unit step signal

Standard Test Signals

- Ramp signal
  - The ramp signal imitate the constant velocity characteristic of actual input signal.

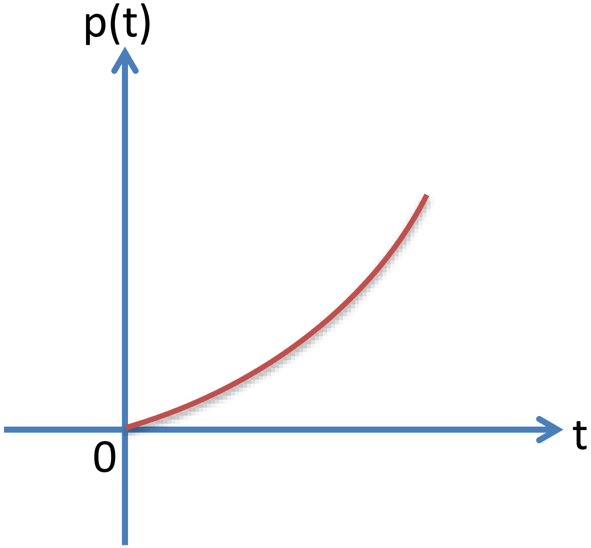


$$r(t) = \begin{cases} At & t \geq 0 \\ 0 & t < 0 \end{cases}$$

  - If *A=1*, the ramp signal is called unit ramp signal

Standard Test Signals

- Parabolic signal
  - The parabolic signal imitate the constant acceleration characteristic of actual input signal.

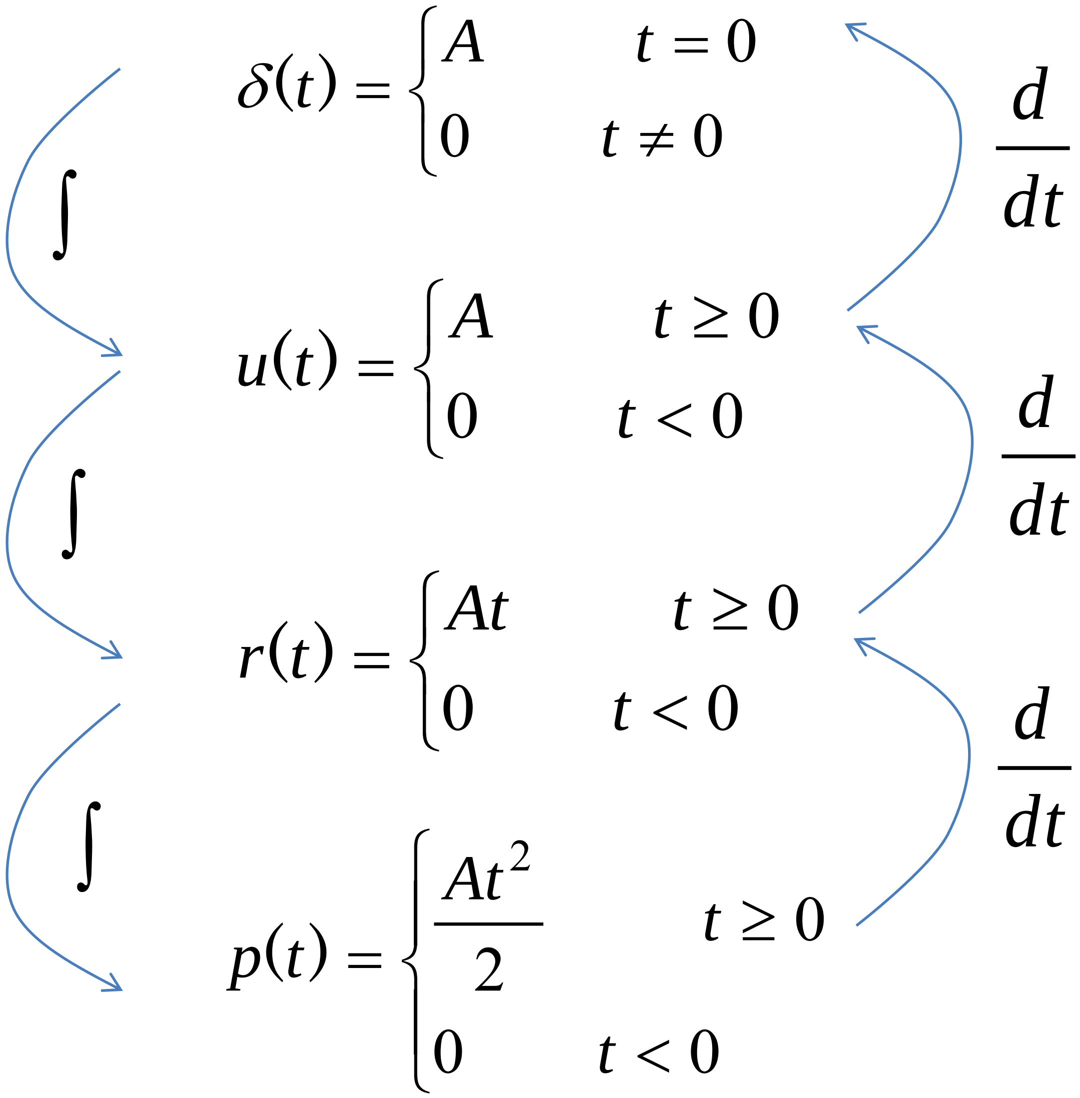


$$p(t) = \begin{cases} \dfrac{At^2}{2} & t \geq 0 \\ 0 & t < 0 \end{cases}$$

  - If *A=1*, the parabolic signal is called unit parabolic signal.

Relation between standard Test Signals

- Impulse
- Step
- Ramp
- Parabolic



$$\delta(t) = \begin{cases} A & t = 0 \\ 0 & t \neq 0 \end{cases}$$

$$u(t) = \begin{cases} A & t \geq 0 \\ 0 & t < 0 \end{cases}$$

$$r(t) = \begin{cases} At & t \geq 0 \\ 0 & t < 0 \end{cases}$$

$$p(t) = \begin{cases} \dfrac{At^2}{2} & t \geq 0 \\ 0 & t < 0 \end{cases}$$

Time Response

- In time domain analysis, time is the independent variable.

- When a system is given an excitation(input) , there is response(output). This response varies with time and it is called time response.

Types:-

Generally the response of any system has two types,

- Transcient Response

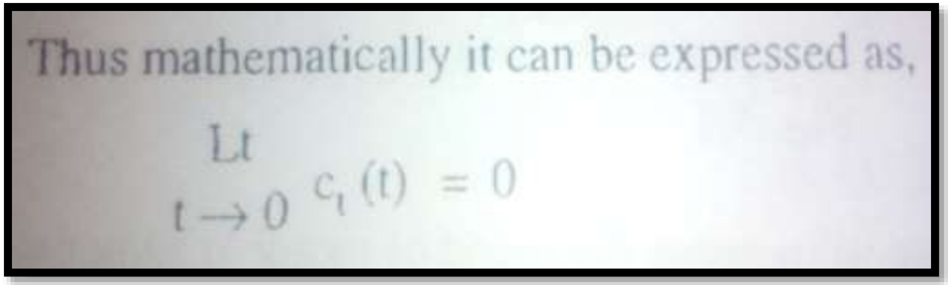
- Steady State Response

Transient Response

- The system takes certain time to achieve its final value.

DEF :- The variation of output response during the time ; it takes to reach its final value is called as transient response of a system.

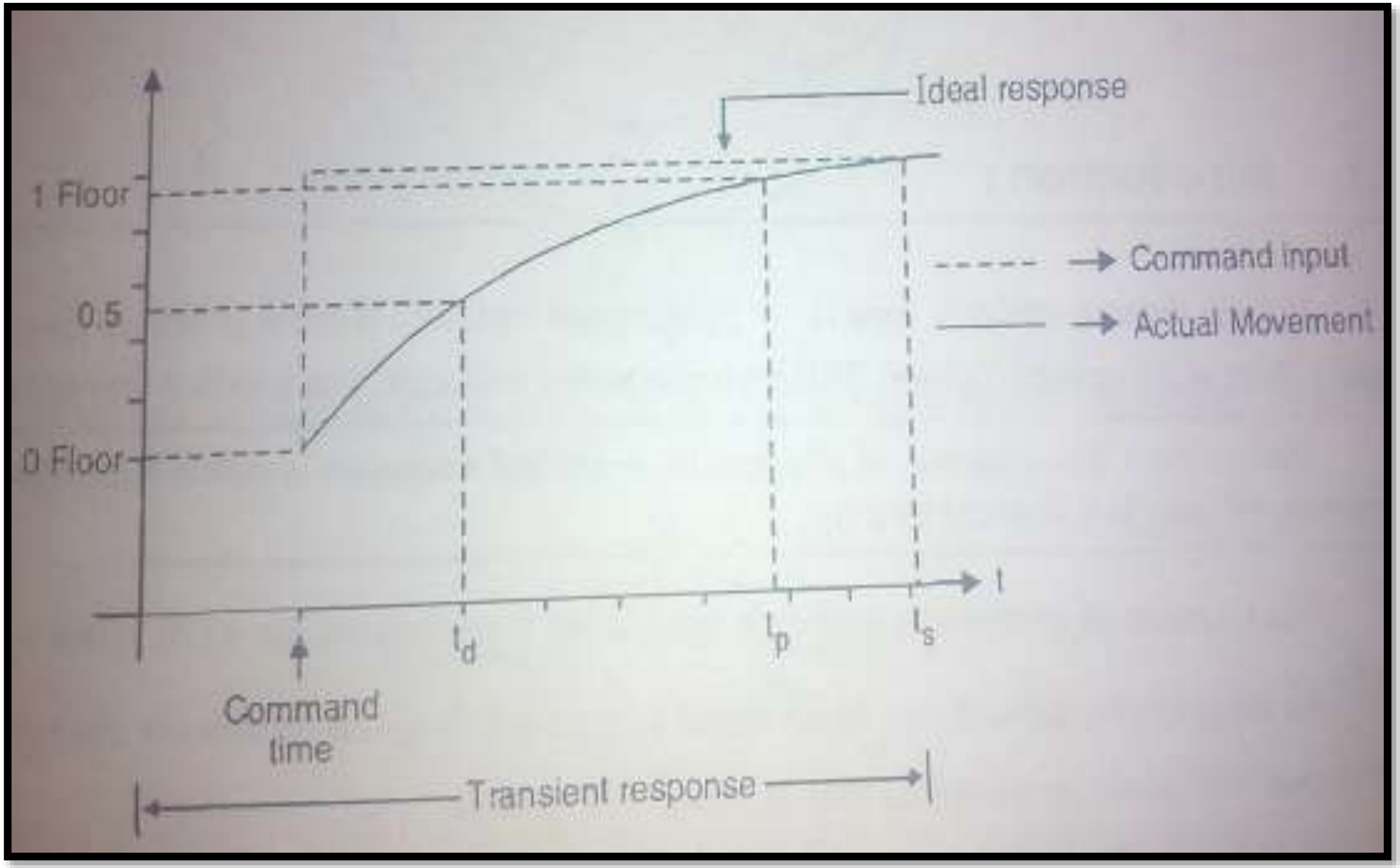
The time period required to reach to its final value is called as transition period.



Thus mathematically it can be expressed as,

$$\underset{t \rightarrow 0}{\mathrm{Lt}} \; c_t(t) = 0$$

Example


Ideal response
1 Floor
0.5
0 Floor
Command input
Actual Movement
t
t_d
t_p
t_s
Command time
Transient response

From the transient response we can know,

- When the system is begin to respond after an input is given.
- How much time it takes to reach the output for the first time.
- Whether the output shoots beyond the desired value and how much.
- Whether the output oscillates about its final value.
- When does it settle to the final value.

Steady State Response

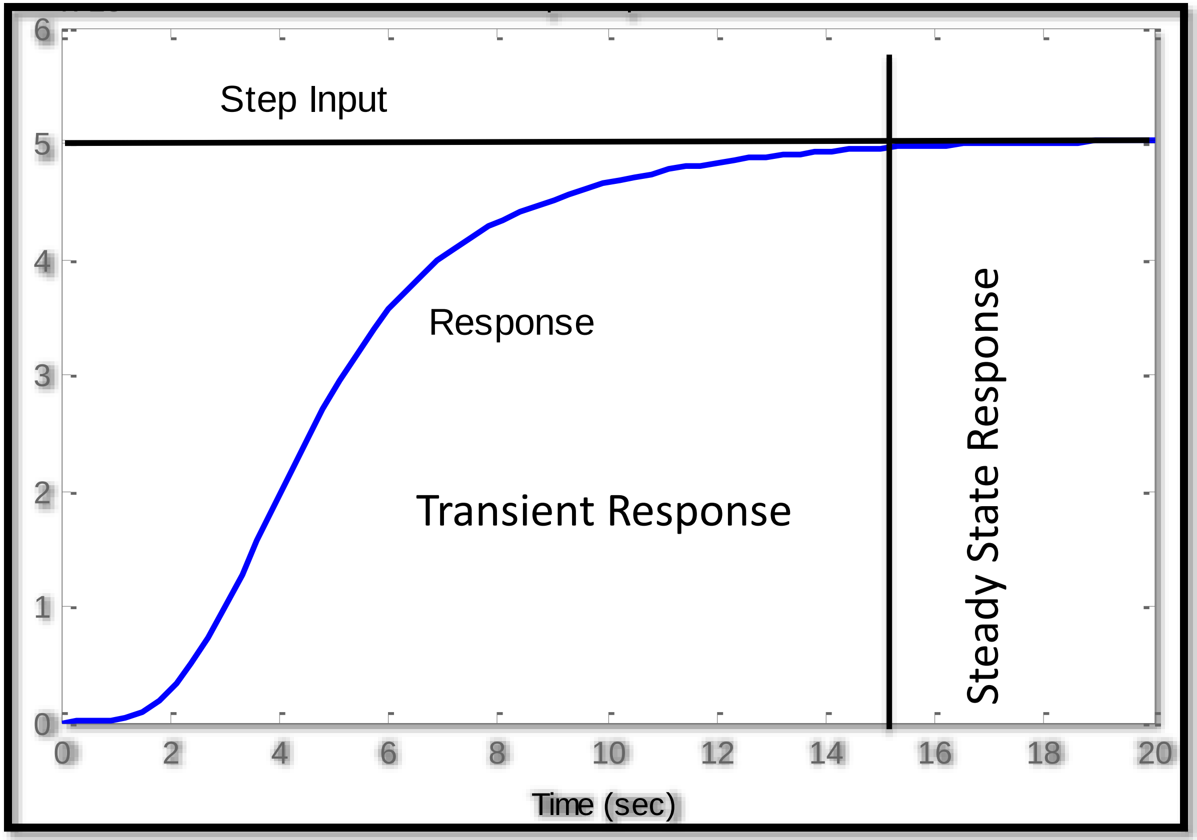
- It is basically the final value achieved by the system output.

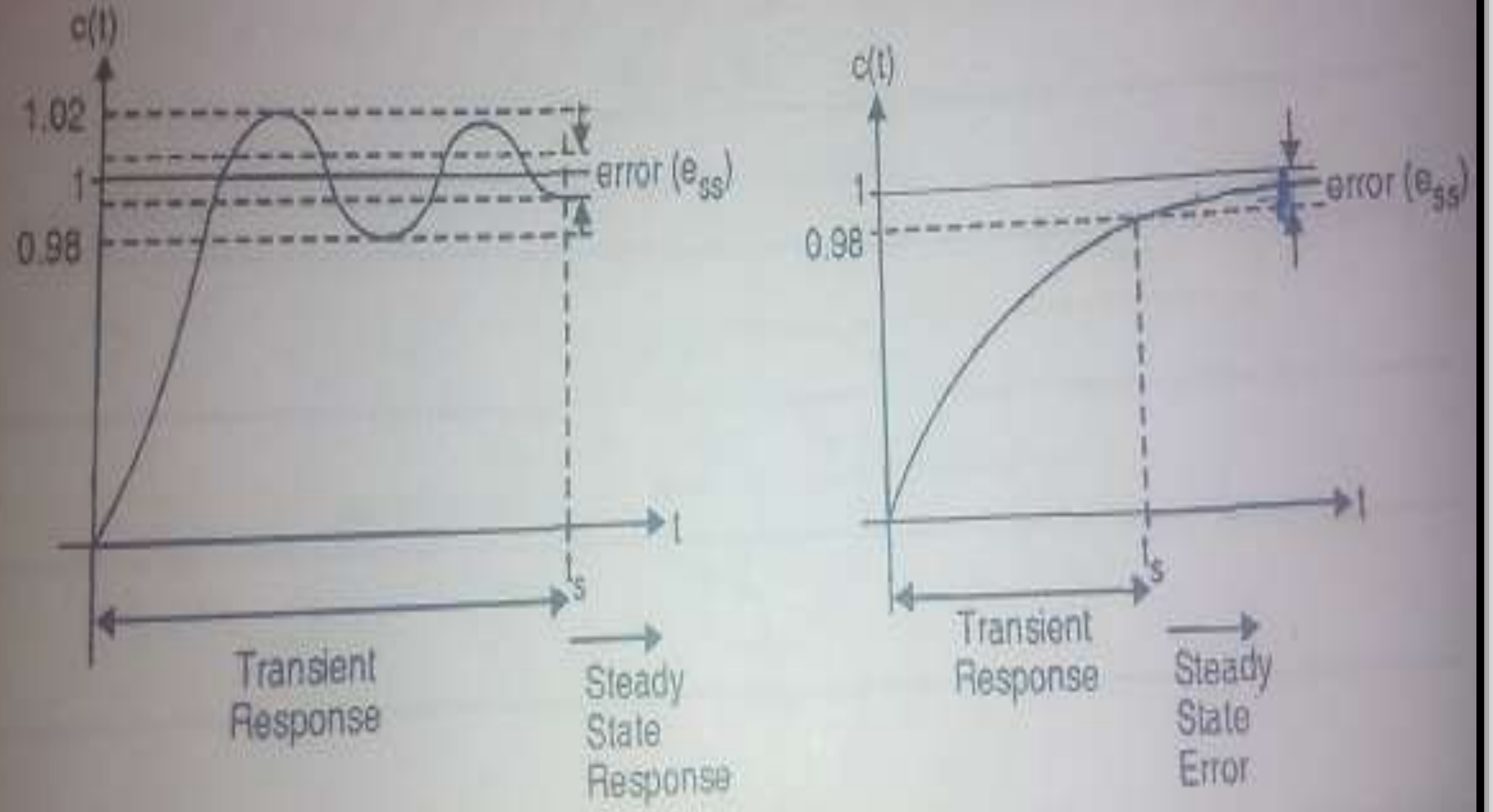
- The steady state response starts when the transient response completely dies out.

DEF :- “ The part of response the remains after the transients have dies out is called as steady state response. ”

From the steady state we can know,

- How long it took before steady state was reached.
- Whether there is any error between the desired and actual values.
- Whether this error is constant , zero or infinite.


Step Input
Response
Transient Response
Steady State Response
0
1
2
3
4
5
6
0
2
4
6
8
10
12
14
16
18
20
Time (sec)


c(t)
1.02
1
0.98
error (e_{ss})
t
t_s
Transient Response
Steady State Response
c(t)
1
0.98
error (e_{ss})
t
t_s
Transient Response
Steady State Error

(a) (b)

Fig. 5.3

Parameters

1. Total Response :

- The total response of system is addition of transient response and steady state response.

- It is denoted by c(t)

2. Steady state error:

- The difference between desired output and actual output of system is called as steady state error.

Analysis of First Order System

- A first-order system without zeros can be represented by the following transfer function

$$\frac{C(s)}{R(s)} = \frac{1}{\tau s + 1}$$

- Given a step input, i.e., *R*(s) = 1/s , then the system output (called **step response** in this case) is

$$C(s) = \frac{1}{\tau s + 1} R(s) = \frac{1}{s(\tau s + 1)} = \frac{1}{s} - \frac{1}{s + \frac{1}{\tau}}$$

Taking inverse Laplace transform , we have the step response

$$\boxed{c(t) = 1 - e^{-\frac{t}{\tau}}}$$

Time Constant: If t=τ, So the step response is

$C(\tau) = (1- 0.37) = 0.63$

τ is referred to as the **time constant** of the response.

Analysis of second order system

- Damping Ratio :

  The damping is measured by a factor called as damping ratio.

- It is denoted by ζ .

- So when zeta is maximum ; it produces maximum opposition to the oscillatory behavior of system.

- Natural frequency of oscillation :

When zeta is zero ; that means there is no opposition to the oscillatory behavior of a system then the system will oscillate naturally.

- Thus when zeta is zero the system oscillates with max frequency.

- A general second-order system is characterized by the following transfer function:

$$G(s) = \frac{b}{s^2 + as + b}$$

- We can re-write the above transfer function in the following form (closed loop transfer function):

$$\boxed{G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}}$$

According the value of ζ, a second-order system can be set into one of the four categories:

1. Over damped :

when the system has two real distinct poles (ζ >1).

2. Under damped :

when the system has two complex conjugate poles (0 <ζ <1)

3. Un-damped :

when the system has two imaginary poles (ζ = 0).

4. Critically damped :

when the system has two real but equal poles (ζ = 1).

THANK YOU